

$U(2, 2) \rightarrow U(2) \times U(2)$  decompositions

The branching rules given below all involve irreducible representations of the form  $\{2(\overline{m}; m)\}$  where  $m$  is a nonnegative integer where the notation follows King and Wybourne (J.Phys. A:Math.Gen. **18**,3113-3139 (1985)). While these decompositions have been evaluated for  $U(2, 2) \rightarrow U(2) \times U(2)$  they in fact hold without modification for  $U(p, q) \rightarrow U(q) \times U(p)$  when  $p, q \geq 2$ . These were all evaluated using standard commands in SCHUR. These branching rules are relevant to the two-electron problem. The discrete states of a single electron in a Coulomb field belong to the irrep  $\{1(\overline{0}; 0)\}$  of  $SU(2, 2)$  while for two non-interacting electrons one may resolve the Kronecker square as

$$\{1(\overline{0}; 0)\} \otimes \{2\} = \sum_{m=0}^{\infty} \{2(\overline{2m}; 2m)\} \quad S = 0$$

$$\{1(\overline{0}; 0)\} \otimes \{1^2\} = \sum_{m=0}^{\infty} \{2(\overline{2m+1}; 2m+1)\} \quad S = 1$$

$\{2(\overline{0}; 0)\} \rightarrow$			
$\{\overline{0}\} \times \{0\}$	$+ \{\overline{1}\} \times \{1\}$	$+ \{\overline{1^2}\} \times \{1^2\}$	$+ \{\overline{2}\} \times \{2\}$
$+ \{\overline{21}\} \times \{21\}$	$+ \{\overline{2^2}\} \times \{2^2\}$	$+ \{\overline{3}\} \times \{3\}$	$+ \{\overline{31}\} \times \{31\}$
$+ \{\overline{32}\} \times \{32\}$	$+ \{\overline{3^2}\} \times \{3^2\}$	$+ \{\overline{4}\} \times \{4\}$	$+ \{\overline{41}\} \times \{41\}$
$+ \{\overline{42}\} \times \{42\}$	$+ \{\overline{43}\} \times \{43\}$	$+ \{\overline{4^2}\} \times \{4^2\}$	$+ \{\overline{5}\} \times \{5\}$
$+ \{\overline{51}\} \times \{51\}$	$+ \{\overline{52}\} \times \{52\}$	$+ \{\overline{53}\} \times \{53\}$	$+ \{\overline{54}\} \times \{54\}$
$+ \{\overline{5^2}\} \times \{5^2\}$	$+ \{\overline{6}\} \times \{6\}$	$+ \{\overline{61}\} \times \{61\}$	$+ \{\overline{62}\} \times \{62\}$
$+ \{\overline{63}\} \times \{63\}$	$+ \{\overline{64}\} \times \{64\}$	$+ \{\overline{7}\} \times \{7\}$	$+ \{\overline{71}\} \times \{71\}$
$+ \{\overline{72}\} \times \{72\}$	$+ \{\overline{73}\} \times \{73\}$	$+ \{\overline{8}\} \times \{8\}$	$+ \{\overline{81}\} \times \{81\}$
$+ \{\overline{82}\} \times \{82\}$	$+ \{\overline{9}\} \times \{9\}$	$+ \{\overline{91}\} \times \{91\}$	$+ \{\overline{10}\} \times \{10\}$
$\{2(\overline{1}; 1)\} \rightarrow$			
$\{\overline{1}\} \times \{1\}$	$+ \{\overline{1^2}\} \times \{2\}$	$+ \{\overline{2}\} \times \{1^2\}$	$+ \{\overline{2}\} \times \{2\}$
$+ \{\overline{21}\} \times \{21\}$	$+ \{\overline{21}\} \times \{3\}$	$+ \{\overline{2^2}\} \times \{31\}$	$+ \{\overline{3}\} \times \{21\}$
$+ \{\overline{3}\} \times \{3\}$	$+ \{\overline{31}\} \times \{2^2\}$	$+ \{\overline{31}\} \times \{31\}$	$+ \{\overline{31}\} \times \{4\}$
$+ \{\overline{32}\} \times \{32\}$	$+ \{\overline{32}\} \times \{41\}$	$+ \{\overline{3^2}\} \times \{42\}$	$+ \{\overline{4}\} \times \{31\}$
$+ \{\overline{4}\} \times \{4\}$	$+ \{\overline{41}\} \times \{32\}$	$+ \{\overline{41}\} \times \{41\}$	$+ \{\overline{41}\} \times \{5\}$
$+ \{\overline{42}\} \times \{3^2\}$	$+ \{\overline{42}\} \times \{42\}$	$+ \{\overline{42}\} \times \{51\}$	$+ \{\overline{43}\} \times \{43\}$
$+ \{\overline{43}\} \times \{52\}$	$+ \{\overline{4^2}\} \times \{53\}$	$+ \{\overline{5}\} \times \{41\}$	$+ \{\overline{5}\} \times \{5\}$
$+ \{\overline{51}\} \times \{42\}$	$+ \{\overline{51}\} \times \{51\}$	$+ \{\overline{51}\} \times \{6\}$	$+ \{\overline{52}\} \times \{43\}$
$+ \{\overline{52}\} \times \{52\}$	$+ \{\overline{52}\} \times \{61\}$	$+ \{\overline{53}\} \times \{4^2\}$	$+ \{\overline{53}\} \times \{53\}$
$+ \{\overline{53}\} \times \{62\}$	$+ \{\overline{54}\} \times \{54\}$	$+ \{\overline{54}\} \times \{63\}$	$+ \{\overline{5^2}\} \times \{64\}$
$+ \{\overline{6}\} \times \{51\}$	$+ \{\overline{6}\} \times \{6\}$	$+ \{\overline{61}\} \times \{52\}$	$+ \{\overline{61}\} \times \{61\}$
$+ \{\overline{61}\} \times \{7\}$	$+ \{\overline{62}\} \times \{53\}$	$+ \{\overline{62}\} \times \{62\}$	$+ \{\overline{62}\} \times \{71\}$
$+ \{\overline{63}\} \times \{54\}$	$+ \{\overline{63}\} \times \{63\}$	$+ \{\overline{63}\} \times \{72\}$	$+ \{\overline{64}\} \times \{5^2\}$
$+ \{\overline{64}\} \times \{64\}$	$+ \{\overline{64}\} \times \{73\}$	$+ \{\overline{7}\} \times \{61\}$	$+ \{\overline{7}\} \times \{7\}$
$+ \{\overline{71}\} \times \{62\}$	$+ \{\overline{71}\} \times \{71\}$	$+ \{\overline{71}\} \times \{8\}$	$+ \{\overline{72}\} \times \{63\}$
$+ \{\overline{72}\} \times \{72\}$	$+ \{\overline{72}\} \times \{81\}$	$+ \{\overline{73}\} \times \{64\}$	$+ \{\overline{73}\} \times \{73\}$
$+ \{\overline{73}\} \times \{82\}$	$+ \{\overline{8}\} \times \{71\}$	$+ \{\overline{8}\} \times \{8\}$	$+ \{\overline{81}\} \times \{72\}$
$+ \{\overline{81}\} \times \{81\}$	$+ \{\overline{81}\} \times \{9\}$	$+ \{\overline{82}\} \times \{73\}$	$+ \{\overline{82}\} \times \{82\}$
$+ \{\overline{82}\} \times \{91\}$	$+ \{\overline{9}\} \times \{81\}$	$+ \{\overline{9}\} \times \{9\}$	$+ \{\overline{91}\} \times \{82\}$
$+ \{\overline{91}\} \times \{91\}$	$+ \{\overline{9}\} \times \{81\}$	$+ \{\overline{9}\} \times \{9\}$	$+ \{\overline{91}\} \times \{82\}$





$\{2(\overline{7}; 7)\} \rightarrow$			
$\{\overline{7}\} \times \{7\}$	$+ \{\overline{71}\} \times \{8\}$	$+ \{\overline{72}\} \times \{9\}$	$+ \{\overline{73}\} \times \{10\}$
$+ \{\overline{8}\} \times \{71\}$	$+ \{\overline{8}\} \times \{8\}$	$+ \{\overline{81}\} \times \{81\}$	$+ \{\overline{81}\} \times \{9\}$
$+ \{\overline{82}\} \times \{91\}$	$+ \{\overline{82}\} \times \{10\}$	$+ \{\overline{9}\} \times \{72\}$	$+ \{\overline{9}\} \times \{81\}$
$+ \{\overline{9}\} \times \{9\}$	$+ \{\overline{91}\} \times \{82\}$	$+ \{\overline{91}\} \times \{91\}$	$+ \{\overline{91}\} \times \{10\}$
$+ \{\overline{10}\} \times \{73\}$	$+ \{\overline{10}\} \times \{82\}$	$+ \{\overline{10}\} \times \{91\}$	$+ \{\overline{10}\} \times \{10\}$
$\{2(\overline{8}; 8)\} \rightarrow$			
$\{\overline{8}\} \times \{8\}$	$+ \{\overline{81}\} \times \{9\}$	$+ \{\overline{82}\} \times \{10\}$	$+ \{\overline{9}\} \times \{81\}$
$+ \{\overline{9}\} \times \{9\}$	$+ \{\overline{91}\} \times \{91\}$	$+ \{\overline{91}\} \times \{10\}$	$+ \{\overline{10}\} \times \{82\}$
$+ \{\overline{10}\} \times \{91\}$	$+ \{\overline{10}\} \times \{10\}$		
$\{2(\overline{9}; 9)\} \rightarrow$			
$\{\overline{9}\} \times \{9\}$	$+ \{\overline{91}\} \times \{10\}$	$+ \{\overline{10}\} \times \{91\}$	$+ \{\overline{10}\} \times \{10\}$