Notes on Plethysms in $SO^*(2n)$

We label the irreps as $[k(\lambda)]$ and note that under $SO^*(2n) \to U(n)$ we have

$$[k(\lambda)] \to \varepsilon^k \cdot \{(\{\lambda_s\}^{2k} \cdot B_{2k})\}_{2k,n} \tag{1}$$

where $\{\lambda_s\}^{2k}$ is the signed sequence of λ evaluated in $Sp(2k)$.

The Kronecker product of a pair of irreps of $SO^*(2n)$ may be evaluated as

$$[k(\lambda)] \times [\ell(\mu)] = [k + \ell(\{\lambda_s\}^{2k} \cdot \{\mu_s\}^{2\ell} \cdot B_{k+\ell})_N] \tag{2}$$

where $N = min(k + \ell, n)$ and B is the infinite $S-$function series

$$B = \sum_{\beta} \{\beta\}$$

where the summation is over all partitions (β) whose parts are repeated an even number of times.

There is an infinite set of fundamental unirreps of $SO^*(2n)$ which we label as $[1(m)]$ with $m = 0, 1, 2, \ldots$. Use of Eq. (2) leads to

$$[1(m)] \times [1(m')] = \sum_{p=0}^{\infty} \sum_{x=0}^{m'} [2(2(m + m') + p - x, p + x)] \ (m \geq m') \tag{3}$$

The Kronecker squares of the fundamental unirreps may be resolved into their symmetric and antisymmetric parts following Thibon, Toumazet and Wybourne[1] to give

$$[1(m)] \otimes \{2\} = \sum_{p=0}^{\infty} \sum_{x=0}^{m'} [2(2m + p - x, p + x)] \ (p + x \ even) \tag{4a}$$

$$[1(m)] \otimes \{1^2\} = \sum_{p=0}^{\infty} \sum_{x=0}^{m'} [2(2m + p - x, p + x)] \ (p + x \ odd) \tag{4b}$$

Equations (4a) and (4b) are in fact special cases of the general result which follows from the adaption of a result given earlier[2] for $Sp(2n, R)$.

$$[k(\lambda)] \otimes \{2\} = [2k(\{\lambda_s\}^{2k} \otimes \{2\} \cdot B_+)_N] + [2k(\{\lambda_s\}^{2k} \otimes \{1^2\} \cdot B_-)_N] \tag{5a}$$

$$[k(\lambda)] \otimes \{1^2\} = [2k(\{\lambda_s\}^{2k} \otimes \{1^2\} \cdot B_+)_N] + [2k(\{\lambda_s\}^{2k} \otimes \{2\} \cdot B_-)_N] \tag{5b}$$

for resolving the Kronecker square of an arbitrary unirrep $[k(\lambda)]$ of $SO^*(2n)$ where

$$B_{\pm} = \{1^2\} \otimes M_{\pm}$$

with $M_{\pm}$ being the infinite series of $S-$functions involving partitions (m) of just one part with M_+ (M_-) being associated with *even (odd)* values of the integer m.

References

1. Thibon J-Y, Toumazet F and Wybourne B G , *J. Phys. A: Math. Gen.* (in press) (1997).
2. King R C and Wybourne B G , *J. Phys. A: Math. Gen.* ??