Resolution of the squares and cubes of the fundamentals of $SO^*(6)$

We write the set of fundamental irreps of $SO^*(6)$ as

$$H = \sum_{m=0}^{\infty} [1(m)]$$

Below we give, to terms up to weight 12, the resolutions of the square and cube of H.

$H \otimes \{2\}$

[2(0)]	+ [2(1)]	+ 2[2(2)]	+ 2[2(21)]	+ 3[2(2^2)]
+ 2[2(3)]	+ 2[2(31)]	+ 3[2(32)]	+ 3[2(4)]	+ 4[2(41)]
+ 6[2(42)]	+ 4[2(43)]	+ 5[2(4^2)]	+ 3[2(5)]	+ 4[2(51)]
+ 6[2(52)]	+ 4[2(53)]	+ 5[2(54)]	+ 4[2(6)]	+ 6[2(61)]
+ 9[2(62)]	+ 8[2(63)]	+ 10[2(64)]	+ 6[2(65)]	+ 7[2(6^2)]
+ 4[2(7)]	+ 6[2(71)]	+ 9[2(72)]	+ 8[2(73)]	+ 10[2(74)]
+ 6[2(75)]	+ 5[2(8)]	+ 8[2(81)]	+ 12[2(82)]	+ 12[2(83)]
+ 15[2(84)]	+ 5[2(9)]	+ 8[2(91)]	+ 12[2(92)]	+ 12[2(93)]
+ 6[2(10)]	+ 10[2(10 1)]	+ 15[2(10 2)]	+ 6[2(11)]	+ 10[2(11 1)]
+ 7[2(12)]				

$H \otimes \{1^2\}$

[2(1)]	+ 2[2(1^2)]	+ [2(2)]	+ 2[2(21)]	+ 2[2(3)]
+ 4[2(31)]	+ 3[2(32)]	+ 4[2(3^2)]	+ 2[2(4)]	+ 4[2(41)]
+ 3[2(42)]	+ 4[2(43)]	+ 3[2(5)]	+ 6[2(51)]	+ 6[2(52)]
+ 8[2(53)]	+ 5[2(54)]	+ 6[2(5^2)]	+ 3[2(6)]	+ 6[2(61)]
+ 6[2(62)]	+ 8[2(63)]	+ 5[2(64)]	+ 6[2(65)]	+ 4[2(7)]
+ 8[2(71)]	+ 9[2(72)]	+ 12[2(73)]	+ 10[2(74)]	+ 12[2(75)]
+ 4[2(8)]	+ 8[2(81)]	+ 9[2(82)]	+ 12[2(83)]	+ 10[2(84)]
+ 5[2(9)]	+ 10[2(91)]	+ 12[2(92)]	+ 16[2(93)]	+ 5[2(10)]
+ 10[2(10 1)]	+ 12[2(10 2)]	+ 6[2(11)]	+ 12[2(11 1)]	+ 6[2(12)]

From the above results one can conjecture that

$$H \otimes \{2\} = \sum_{x=0}^{\infty} \sum_{m=2x}^{\infty} (2x+1) \left(\left[\frac{m}{2} \right] - x + 1 \right) [2(m, 2x)]$$
$$+ \sum_{x=1}^{\infty} \sum_{m=2x}^{\infty} 2x \left(\left[\frac{m}{2} \right] - x + 1 \right) [2(m, 2x-1)] \qquad (1a)$$

$$H \otimes \{1^2\} = \sum_{x=0}^{\infty} \sum_{m=2x+1}^{\infty} \left((2x+1) \left[\frac{m-1}{2} \right] - x^2 + 1 \right) [2(m, 2x)]$$
$$+ \sum_{x=1}^{\infty} \sum_{m=2x-1}^{\infty} 2x \left(\left[\frac{m-1}{2} \right] - x + 2 \right) [2(m, 2x-1)] \qquad (1b)$$

$H \otimes \{3\}$

$[3(0)]$	$+\ [3(1)]$	$+\ 2[3(2)]$	$+\ 2[3(21)]$	$+\ [3(21^2)]$
$+\ 4[3(2^2)]$	$+\ 4[3(2^21)]$	$+\ 4[3(2^3)]$	$+\ 3[3(3)]$	$+\ 4[3(31)]$
$+\ 2[3(31^2)]$	$+\ 8[3(32)]$	$+\ 8[3(321)]$	$+\ 8[3(32^2)]$	$+\ 5[3(3^2)]$
$+\ 5[3(3^21)]$	$+\ 5[3(3^22)]$	$+\ [3(3^3)]$	$+\ 4[3(4)]$	$+\ 7[3(41)]$
$+\ 5[3(41^2)]$	$+\ 15[3(42)]$	$+\ 18[3(421)]$	$+\ 17[3(42^2)]$	$+\ 15[3(43)]$
$+\ 20[3(431)]$	$+\ 20[3(432)]$	$+\ 7[3(43^2)]$	$+\ 14[3(4^2)]$	$+\ 22[3(4^21)]$
$+\ 24[3(4^22)]$	$+\ 16[3(4^23)]$	$+\ 11[3(4^3)]$	$+\ 5[3(5)]$	$+\ 10[3(51)]$
$+\ 8[3(51^2)]$	$+\ 22[3(52)]$	$+\ 28[3(521)]$	$+\ 26[3(52^2)]$	$+\ 26[3(53)]$
$+\ 37[3(531)]$	$+\ 38[3(532)]$	$+\ 16[3(53^2)]$	$+\ 30[3(54)]$	$+\ 48[3(541)]$
$+\ 54[3(542)]$	$+\ 38[3(543)]$	$+\ 18[3(5^2)]$	$+\ 29[3(5^21)]$	$+\ 34[3(5^22)]$
$+\ 7[3(6)]$	$+\ 15[3(61)]$	$+\ 12[3(61^2)]$	$+\ 33[3(62)]$	$+\ 42[3(621)]$
$+\ 39[3(62^2)]$	$+\ 44[3(63)]$	$+\ 63[3(631)]$	$+\ 67[3(632)]$	$+\ 33[3(63^2)]$
$+\ 57[3(64)]$	$+\ 90[3(641)]$	$+\ 105[3(642)]$	$+\ 52[3(65)]$	$+\ 83[3(651)]$
$+\ 41[3(6^2)]$	$+\ 8[3(7)]$	$+\ 19[3(71)]$	$+\ 17[3(71^2)]$	$+\ 43[3(72)]$
$+\ 58[3(721)]$	$+\ 53[3(72^2)]$	$+\ 61[3(73)]$	$+\ 92[3(731)]$	$+\ 98[3(732)]$
$+\ 84[3(74)]$	$+\ 138[3(741)]$	$+\ 88[3(75)]$	$+\ 10[3(8)]$	$+\ 25[3(81)]$
$+\ 23[3(81^2)]$	$+\ 57[3(82)]$	$+\ 78[3(821)]$	$+\ 71[3(82^2)]$	$+\ 85[3(83)]$
$+\ 130[3(831)]$	$+\ 122[3(84)]$	$+\ 12[3(9)]$	$+\ 31[3(91)]$	$+\ 29[3(91^2)]$
$+\ 71[3(92)]$	$+\ 98[3(921)]$	$+\ 110[3(93)]$	$+\ 14[3(10\ )]$	$+\ 38[3(10\ 1)]$
$+\ 37[3(10\ 1^2)]$	$+\ 88[3(10\ 2)]$	$+\ 16[3(11\ )]$	$+\ 45[3(11\ 1)]$	$+\ 19[3(12\ )]$

$H \otimes \{21\}$

$[3(1)]$	$+\ 2[3(1^2)]$	$+\ [3(1^3)]$	$+\ 2[3(2)]$	$+\ 5[3(21)]$
$+\ 4[3(21^2)]$	$+\ 5[3(2^2)]$	$+\ 6[3(2^21)]$	$+\ 3[3(2^3)]$	$+\ 3[3(3)]$
$+\ 9[3(31)]$	$+\ 9[3(31^2)]$	$+\ 13[3(32)]$	$+\ 18[3(321)]$	$+\ 11[3(32^2)]$
$+\ 11[3(3^2)]$	$+\ 17[3(3^21)]$	$+\ 14[3(3^22)]$	$+\ 6[3(3^3)]$	$+\ 5[3(4)]$
$+\ 15[3(41)]$	$+\ 15[3(41^2)]$	$+\ 25[3(42)]$	$+\ 34[3(421)]$	$+\ 23[3(42^2)]$
$+\ 30[3(43)]$	$+\ 45[3(431)]$	$+\ 41[3(432)]$	$+\ 22[3(43^2)]$	$+\ 24[3(4^2)]$
$+\ 36[3(4^21)]$	$+\ 37[3(4^22)]$	$+\ 27[3(4^23)]$	$+\ 11[3(4^3)]$	$+\ 7[3(5)]$
$+\ 22[3(51)]$	$+\ 23[3(51^2)]$	$+\ 40[3(52)]$	$+\ 56[3(521)]$	$+\ 40[3(52^2)]$
$+\ 55[3(53)]$	$+\ 85[3(531)]$	$+\ 81[3(532)]$	$+\ 47[3(53^2)]$	$+\ 59[3(54)]$
$+\ 94[3(541)]$	$+\ 100[3(542)]$	$+\ 78[3(543)]$	$+\ 42[3(5^2)]$	$+\ 70[3(5^21)]$
$+\ 78[3(5^22)]$	$+\ 9[3(6)]$	$+\ 30[3(61)]$	$+\ 33[3(61^2)]$	$+\ 58[3(62)]$
$+\ 84[3(621)]$	$+\ 62[3(62^2)]$	$+\ 86[3(63)]$	$+\ 137[3(631)]$	$+\ 134[3(632)]$
$+\ 81[3(63^2)]$	$+\ 105[3(64)]$	$+\ 174[3(641)]$	$+\ 190[3(642)]$	$+\ 102[3(65)]$
$+\ 176[3(651)]$	$+\ 69[3(6^2)]$	$+\ 12[3(7)]$	$+\ 40[3(71)]$	$+\ 44[3(71^2)]$
$+\ 80[3(72)]$	$+\ 116[3(721)]$	$+\ 88[3(72^2)]$	$+\ 125[3(73)]$	$+\ 200[3(731)]$
$+\ 201[3(732)]$	$+\ 164[3(74)]$	$+\ 274[3(741)]$	$+\ 182[3(75)]$	$+\ 15[3(8)]$
$+\ 51[3(81)]$	$+\ 57[3(81^2)]$	$+\ 105[3(82)]$	$+\ 154[3(821)]$	$+\ 119[3(82^2)]$
$+\ 170[3(83)]$	$+\ 275[3(831)]$	$+\ 234[3(84)]$	$+\ 18[3(9)]$	$+\ 63[3(91)]$
$+\ 72[3(91^2)]$	$+\ 133[3(92)]$	$+\ 198[3(921)]$	$+\ 221[3(93)]$	$+\ 22[3(10\ )]$
$+\ 77[3(10\ 1)]$	$+\ 88[3(10\ 1^2)]$	$+\ 165[3(10\ 2)]$	$+\ 26[3(11\ )]$	$+\ 92[3(11\ 1)]$
$+\ 30[3(12\ )]$				

$H \otimes \{1^3\}$

$[3(1^2)]$	$+\ 2[3(1^3)]$	$+\ 2[3(21)]$	$+\ 4[3(21^2)]$	$+\ [3(2^2)]$
$+\ 2[3(2^21)]$	$+\ [3(3)]$	$+\ 5[3(31)]$	$+\ 7[3(31^2)]$	$+\ 6[3(32)]$
$+\ 8[3(321)]$	$+\ 3[3(32^2)]$	$+\ 8[3(3^2)]$	$+\ 11[3(3^21)]$	$+\ 9[3(3^22)]$
$+\ 7[3(3^3)]$	$+\ [3(4)]$	$+\ 7[3(41)]$	$+\ 11[3(41^2)]$	$+\ 10[3(42)]$
$+\ 16[3(421)]$	$+\ 7[3(42^2)]$	$+\ 15[3(43)]$	$+\ 25[3(431)]$	$+\ 20[3(432)]$
$+\ 15[3(43^2)]$	$+\ 8[3(4^2)]$	$+\ 16[3(4^21)]$	$+\ 13[3(4^22)]$	$+\ 10[3(4^23)]$
$+\ 2[3(4^3)]$	$+\ 2[3(5)]$	$+\ 11[3(51)]$	$+\ 16[3(51^2)]$	$+\ 18[3(52)]$
$+\ 28[3(521)]$	$+\ 15[3(52^2)]$	$+\ 29[3(53)]$	$+\ 48[3(531)]$	$+\ 42[3(532)]$
$+\ 31[3(53^2)]$	$+\ 27[3(54)]$	$+\ 48[3(541)]$	$+\ 46[3(542)]$	$+\ 38[3(543)]$
$+\ 24[3(5^2)]$	$+\ 41[3(5^21)]$	$+\ 45[3(5^22)]$	$+\ 3[3(6)]$	$+\ 15[3(61)]$
$+\ 21[3(61^2)]$	$+\ 26[3(62)]$	$+\ 40[3(621)]$	$+\ 23[3(62^2)]$	$+\ 44[3(63)]$
$+\ 73[3(631)]$	$+\ 67[3(632)]$	$+\ 51[3(63^2)]$	$+\ 48[3(64)]$	$+\ 84[3(641)]$
$+\ 85[3(642)]$	$+\ 52[3(65)]$	$+\ 90[3(651)]$	$+\ 31[3(6^2)]$	$+\ 4[3(7)]$
$+\ 20[3(71)]$	$+\ 28[3(71^2)]$	$+\ 37[3(72)]$	$+\ 58[3(721)]$	$+\ 36[3(72^2)]$
$+\ 64[3(73)]$	$+\ 108[3(731)]$	$+\ 102[3(732)]$	$+\ 78[3(74)]$	$+\ 138[3(741)]$
$+\ 94[3(75)]$	$+\ 5[3(8)]$	$+\ 25[3(81)]$	$+\ 35[3(81^2)]$	$+\ 48[3(82)]$
$+\ 76[3(821)]$	$+\ 49[3(82^2)]$	$+\ 85[3(83)]$	$+\ 145[3(831)]$	$+\ 110[3(84)]$
$+\ 7[3(9)]$	$+\ 32[3(91)]$	$+\ 43[3(91^2)]$	$+\ 63[3(92)]$	$+\ 98[3(921)]$
$+\ 113[3(93)]$	$+\ 8[3(10\ )]$	$+\ 38[3(10\ 1)]$	$+\ 52[3(10\ 1^2)]$	$+\ 77[3(10\ 2)]$
$+\ 10[3(11\ )]$	$+\ 46[3(11\ 1)]$	$+\ 12[3(12\ )]$		

H^2

[2(0)]	+ 2[2(1)]	+ 2[2(1^2)]	+ 3[2(2)]	+ 4[2(21)]
+ 3[2(2^2)]	+ 4[2(3)]	+ 6[2(31)]	+ 6[2(32)]	+ 4[2(3^2)]
+ 5[2(4)]	+ 8[2(41)]	+ 9[2(42)]	+ 8[2(43)]	+ 5[2(4^2)]
+ 6[2(5)]	+ 10[2(51)]	+ 12[2(52)]	+ 12[2(53)]	+ 10[2(54)]
+ 6[2(5^2)]	+ 7[2(6)]	+ 12[2(61)]	+ 15[2(62)]	+ 16[2(63)]
+ 15[2(64)]	+ 12[2(65)]	+ 7[2(6^2)]	+ 8[2(7)]	+ 14[2(71)]
+ 18[2(72)]	+ 20[2(73)]	+ 20[2(74)]	+ 18[2(75)]	+ 9[2(8)]
+ 16[2(81)]	+ 21[2(82)]	+ 24[2(83)]	+ 25[2(84)]	+ 10[2(9)]
+ 18[2(91)]	+ 24[2(92)]	+ 28[2(93)]	+ 11[2(10)]	+ 20[2(10 1)]
+ 27[2(10 2)]	+ 12[2(11)]	+ 22[2(11 1)]	+ 13[2(12)]	

H^3

[3(0)]	+ 3[3(1)]	+ 5[3(1^2)]	+ 4[3(1^3)]	+ 6[3(2)]
+ 14[3(21)]	+ 13[3(21^2)]	+ 15[3(2^2)]	+ 18[3($2^2 1$)]	+ 10[3(2^3)]
+ 10[3(3)]	+ 27[3(31)]	+ 27[3(31^2)]	+ 40[3(32)]	+ 52[3(321)]
+ 33[3(32^2)]	+ 35[3(3^2)]	+ 50[3($3^2 1$)]	+ 42[3($3^2 2$)]	+ 20[3(3^3)]
+ 15[3(4)]	+ 44[3(41)]	+ 46[3(41^2)]	+ 75[3(42)]	+ 102[3(421)]
+ 70[3(42^2)]	+ 90[3(43)]	+ 135[3(431)]	+ 122[3(432)]	+ 66[3(43^2)]
+ 70[3(4^2)]	+ 110[3($4^2 1$)]	+ 111[3($4^2 2$)]	+ 80[3($4^2 3$)]	+ 35[3(4^3)]
+ 21[3(5)]	+ 65[3(51)]	+ 70[3(51^2)]	+ 120[3(52)]	+ 168[3(521)]
+ 121[3(52^2)]	+ 165[3(53)]	+ 255[3(531)]	+ 242[3(532)]	+ 141[3(53^2)]
+ 175[3(54)]	+ 284[3(541)]	+ 300[3(542)]	+ 232[3(543)]	+ 126[3(5^2)]
+ 210[3($5^2 1$)]	+ 235[3($5^2 2$)]	+ 28[3(6)]	+ 90[3(61)]	+ 99[3(61^2)]
+ 175[3(62)]	+ 250[3(621)]	+ 186[3(62^2)]	+ 260[3(63)]	+ 410[3(631)]
+ 402[3(632)]	+ 246[3(63^2)]	+ 315[3(64)]	+ 522[3(641)]	+ 570[3(642)]
+ 308[3(65)]	+ 525[3(651)]	+ 210[3(6^2)]	+ 36[3(7)]	+ 119[3(71)]
+ 133[3(71^2)]	+ 240[3(72)]	+ 348[3(721)]	+ 265[3(72^2)]	+ 375[3(73)]
+ 600[3(731)]	+ 602[3(732)]	+ 490[3(74)]	+ 824[3(741)]	+ 546[3(75)]
+ 45[3(8)]	+ 152[3(81)]	+ 172[3(81^2)]	+ 315[3(82)]	+ 462[3(821)]
+ 358[3(82^2)]	+ 510[3(83)]	+ 825[3(831)]	+ 700[3(84)]	+ 55[3(9)]
+ 189[3(91)]	+ 216[3(91^2)]	+ 400[3(92)]	+ 592[3(921)]	+ 665[3(93)]
+ 66[3(10)]	+ 230[3(10 1)]	+ 265[3(10 1^2)]	+ 495[3(10 2)]	+ 78[3(11)]
+ 275[3(11 1)]	+ 91[3(12)]			