Summary of results on $SO^*(2n)$

1. $SO^*(2n) \to U(n)$ Branching Rule

The procedure is closely related to Sec.5 of KW(1985). Consider the two group chains

$$Sp(4nk, R) \to SO^*(2n) \times Sp(2k) \to U(n) \times Sp(2k) \tag{5.1'}$$
$$Sp(4nk, R) \to U(2nk) \to U(n) \times U(2k) \to U(n) \times Sp(2k) \tag{5.2'}$$

Consideration of the first gives

$$\tilde{\Delta} \to \sum_{\lambda} [k(\lambda)] \times <\lambda> \to \sum_{\lambda} \sum_{\mu} \varepsilon^k R^{\mu}_{\lambda} \{\mu\} \times <\lambda> \tag{5.3'}$$

while the second leads to

$$\tilde{\Delta} \to \sum_{\nu} \varepsilon^k \{\nu \cdot B\} \times <\nu> \tag{5.4'}$$

where

$$B = \sum_{\beta} \{\beta\}$$

with the summation being over all partitions (β) whose parts are repeated an even number of times. Eventually we come to

$$[k(\lambda)] \to \varepsilon^k \cdot \{\{\lambda_s\}^{2k}_N \cdot B_N\}_N \tag{5.6'}$$

where $N = min(2k, n)$ and

$$\tilde{\lambda}_1 \leq k \quad and \; \tilde{\lambda}_1 \leq n \tag{4.4'}$$

2. $SO^*(2n)$ Kronecker Products

Following KW(1985) (8.10-8.15) we have

$$\begin{aligned} Sp(4nk + 4n\ell, R) &\to Sp(4nk, R) \times Sp(4n\ell, R) \\ &\to Sp(2k) \times SO^*(2n) \times Sp(2\ell) \times SO^*(2n) \\ &\to SO^*(2n) \times Sp(2k) \times Sp(2\ell) \end{aligned} \tag{8.10'}$$

and

$$\begin{aligned} Sp(4nk + 4n\ell, R) &\to SO^*(2n) \times Sp(2k + 2\ell) \\ &\to SO^*(2n) \times Sp(2k) \times Sp(2\ell) \end{aligned} \tag{8.11'}$$

and consequentially

$$\begin{aligned} \tilde{\Delta} \to \tilde{\Delta} \times \tilde{\Delta} &\to \sum_{\mu,\nu} [k(\mu)] \times <\mu> \times [\ell(\nu)] \times <\nu> \\ &\to \sum_{\mu,\nu} [k(\mu)] \times [\ell(\nu)] \times <\mu> \times <\nu> \end{aligned} \tag{8.12'}$$

and

$$\begin{aligned} \tilde{\Delta} &\to \sum_{\lambda} [k + \ell(\lambda)] \times <\lambda> \\ &\to \sum_{\lambda} [k + \ell(\lambda)] \times \sum_{\mu,\nu} R^{\mu\nu}_{\lambda} <\mu> \times <\nu> \end{aligned} \tag{8.13'}$$

where the coefficients $R^{\mu\nu}_{\lambda}$ are the branching coefficients for

$$Sp(2k + 2\ell) \to Sp(2k) \times Sp(2\ell)$$

Thus we can write

$$[k(\mu)] \times [\ell(\nu)] = \sum_{\lambda} R_{\lambda}^{\mu\nu} [k + \ell(\lambda)] \tag{8.15'}$$

noting that under $Sp(2k+2\ell) \to Sp(2k) \times Sp(2\ell)$ we have

$$< \lambda > \to \sum_{\epsilon} < \lambda/\epsilon > \times < \epsilon/B >$$

Continuing analogously to KW(1985) (8.16) we are led to

$$[k(\mu)] \times [\ell(\nu)] = [k + \ell((\{\mu_s\}^{2k} \cdot \{\nu_s\}^{2\ell} \cdot B))_N] \tag{8.18'}$$

where $N = min(k+\ell, n)$ and

$$((\lambda))_N = \begin{cases} (\lambda), & \text{if } \tilde{\lambda}_1 \leq n \ and \ \tilde{\lambda}_1 \leq k+\ell \\ 0, & \text{otherwise} \end{cases}$$

Either Eq.(8.15′) or Eq. (8.18′) may be used to evaluate Kronecker products. Eq. (8.15′) has advantages when a single coefficient $R_{\lambda}^{\mu\nu}$ is required. In that case signed sequences are not needed. Eq. (8.18′) is particularly useful in deriving specific formulae as seen in the next section.

3. Kronecker products for the fundamental unirreps of $SO^*(2n)$

There is an infinite set of fundamental unirreps of $SO^*(2n)$ which we label as $[1(m)]$ with $m = 0, 1, 2, \ldots$. Use of Eq. (8.18*) leads to

$$[1(m)] \times [1(m')] = \sum_{p=0}^{\infty} \sum_{x=0}^{min(m,m')} [2(m + m' + p - x, p + x)] \tag{1}$$

4. Resolution of the squares of unirreps of $SO^*(2n)$

In general we have

$$[k(\lambda)] \otimes \{2\} = [2k(\{\lambda_s\}^{2k} \otimes \{2\} \cdot B_+)_N] + [2k(\{\lambda_s\}^{2k} \otimes \{1^2\} \cdot B_-)_N] \tag{2a}$$

$$[k(\lambda)] \otimes \{1^2\} = [2k(\{\lambda_s\}^{2k} \otimes \{1^2\} \cdot B_+)_N] + [2k(\{\lambda_s\}^{2k} \otimes \{2\} \cdot B_-)_N] \tag{2b}$$

where $B_{\pm} = \{1^2\} \otimes M_{\pm}$.

Specialising the above result leads to

$$[1(m)] \otimes \{2\} = \sum_{p=0}^{\infty} \sum_{x=0}^{m} [2(2m + p - x, p + x)] \quad (p + x \ even) \tag{3a}$$

$$[1(m)] \otimes \{1^2\} = \sum_{p=0}^{\infty} \sum_{x=0}^{m} [2(2m + p - x, p + x)] \quad (p + x \ odd) \tag{3b}$$

It is convenient to write the infinite set of fundamentals of $SO^*(2n)$ as

$$H = \sum_{m=0}^{\infty} [1(m)] \tag{4}$$

Explicit calculation of the plethysms $H \otimes \{2\}$ and $H \otimes \{1^2\}$ suggests that

$$\begin{aligned} H \otimes \{2\} = & \sum_{x=0}^{\infty} \sum_{m=2x}^{\infty} (2x+1) \left(\left[\frac{m}{2} \right] - x + 1 \right) [2(m, 2x)] \\ & + \sum_{x=1}^{\infty} \sum_{m=2x}^{\infty} 2x \left(\left[\frac{m}{2} \right] - x + 1 \right) [2(m, 2x-1)] \end{aligned} \tag{5a}$$

$$\begin{aligned} H \otimes \{1^2\} = & \sum_{x=0}^{\infty} \sum_{m=2x+1}^{\infty} \left((2x+1) \left[\frac{m-1}{2} \right] - x^2 + 1 \right) [2(m, 2x)] \\ & + \sum_{x=1}^{\infty} \sum_{m=2x-1}^{\infty} 2x \left(\left[\frac{m-1}{2} \right] - x + 2 \right) [2(m, 2x-1)] \end{aligned} \tag{5b}$$

These results should follow from Eqs. (1) to (3). This remains to be shown. The tables upon which Eq. (5) is based are at http://www.phys.uni.torun.pl/∼bgw

5. General plethysms in $SO^*(2n)$

Plethysms for an arbitrary unirrep $[k(\lambda)]$ of $SO^*(2n)$ may be evaluated by noting that

$$[k(\lambda)] \otimes \{\mu\} = [k\omega_\mu((((\{\lambda_s\}^{2k} \cdot B_{2k}) \otimes \{\mu\})_M \cdot A_M)_M)] \tag{6}$$

where $M = min(k\omega_\mu, n)$

Specialising Eq. (6) we obtain

$$[k(0)] \otimes \{2\} = [2k(B_+)_{2k}] \tag{7a}$$
$$[k(0)] \otimes \{1^2\} = [2k(B_-)_{2k}] \tag{7b}$$

6. Relations between group chains

Starting with the metaplectic group $Mp(4nk)$ we may relate the decompositions between the non-compact subgroups $SO^*(2n)$ and $Sp(2n,R)$ via the diagram:-

$$\begin{array}{ccccc}
SO^*(2n)\times Sp(2k) & \longleftarrow & Mp(4nk) & \longrightarrow & Sp(2n,R)\times O(2k) \\
\downarrow & & & & \downarrow \\
U(n)\times Sp(2k) & & & & U(n)\times O(2k) \\
\downarrow & & & & \downarrow \\
U(n)\times O(2k) & \longleftarrow & & \longrightarrow & U(n)\times O(2k)
\end{array}$$

The terminal subgroup in each case is $U(n)\times O(2k)$ and we understand that the associativity labels for the $O(2k)$ have been removed.

$$\begin{array}{ccccc}
\sum_\kappa [k(\kappa)] \times < \kappa > & \longleftarrow & \tilde{\Delta} & \longrightarrow & \sum_\lambda < k(\lambda) > \times [\lambda] \\
\downarrow & & & & \downarrow \\
\sum_\kappa [k(\kappa)] \times < \kappa/W > & & & & \sum_\lambda < k(\lambda) > \times [\lambda] \\
\downarrow & & & & \downarrow \\
\sum_\lambda \varepsilon^k \cdot \{\{(\lambda\cdot W)_s\}^{2k} \cdot B\}_{2k} \times [\lambda] & \longleftarrow & & \longrightarrow & \sum_\lambda \varepsilon^k \cdot \{\{(\lambda_s\}^{2k} \cdot D\}_{2k} \times [\lambda]
\end{array}$$

where

$$W = \sum_{p,q} (-1)^q \{p+q, q\} \quad p\ even \tag{8}$$

In general

$$(< k(\lambda) > + < k(\lambda^*) >) \downarrow = [k(\{((\lambda+\lambda^*)\cdot W)_s\}^{2k})] \downarrow \quad if\ \lambda \neq \lambda^* \tag{9a}$$
$$< k(\lambda) > \downarrow = [k(\{(\lambda\cdot W)_s\}^{2k})] \downarrow \quad if\ \lambda \equiv \lambda^* \tag{9b}$$

where the signed sequences are evaluated in $Sp(2k)$ and the down arrow symbolizes decomposition to the $U(n)$ subgroup.

Thus

$$(<1(0)> + <1(0^*)>) \downarrow = \sum_{m=0}^{\infty} [1(2m)] \downarrow \qquad (10a)$$

$$<1(1)>\downarrow = \sum_{m=0}^{\infty} [1(2m+1)] \downarrow \qquad (10b)$$

7. Powers and plethysms of H for $SO^*(2n)$

The results of the previous section lead to an alternative method of computing powers of the infinite set of fundamental unirreps, H, of $SO^*(2n)$. It follows from Eq. (10) that the $U(n)$ content of H and

$$S =<1(0)> + <1(0^*)> + <1(0)> \qquad (11)$$

are equivalent and hence so they must be for powers and plethysms. Noting Eqs. (9) and (4.4′) let us introduce the notation

$$(S^p)_p = \sum_{\mu} g^{\mu} < p(\mu) > \qquad (12)$$

with the understanding that we form the $p-$th power of S and retain only those irreps involving partitions (μ) into not more than $p-$parts. We can then write

$$H^p = \sum_{\mu} g^{\mu} [p((\mu \cdot W))_p] \qquad (13)$$

Likewise if

$$(S \otimes \{\lambda\})_{\omega_\lambda} = \sum_{\mu} g^{\mu} < \omega_\lambda(\mu) > \qquad (14)$$

then

$$H \otimes \{\mu\} = \sum_{\mu} g^{\mu} [\omega_\lambda((\mu \cdot W))_{\omega_\lambda}] \qquad (15)$$

As an example we compute the terms of H^2 to weight 12.

$$[<1(0)> + <1(1)> + <1(1^2)>] \times [<1(0)> + <1(1)> + <1(1^2)>] =$$

$<2(0)>$	$+\,2<2(1)>$	$+\,3<2(1^2)>$	$+\,2<2(1^3)>$
$+\,2<2(2)>$	$+\,4<2(21)>$	$+\,2<2(21^2)>$	$+\,3<2(2^2)>$
$+\,2<2(3)>$	$+\,4<2(31)>$	$+\,2<2(31^2)>$	$+\,4<2(32)>$
$+\,3<2(3^2)>$	$+\,2<2(4)>$	$+\,4<2(41)>$	$+\,2<2(41^2)>$
$+\,4<2(42)>$	$+\,4<2(43)>$	$+\,3<2(4^2)>$	$+\,2<2(5)>$
$+\,4<2(51)>$	$+\,2<2(51^2)>$	$+\,4<2(52)>$	$+\,4<2(53)>$
$+\,4<2(54)>$	$+\,3<2(5^2)>$	$+\,2<2(6)>$	$+\,4<2(61)>$
$+\,2<2(61^2)>$	$+\,4<2(62)>$	$+\,4<2(63)>$	$+\,4<2(64)>$
$+\,4<2(65)>$	$+\,3<2(6^2)>$	$+\,2<2(7)>$	$+\,4<2(71)>$
$+\,2<2(71^2)>$	$+\,4<2(72)>$	$+\,4<2(73)>$	$+\,4<2(74)>$
$+\,4<2(75)>$	$+\,2<2(8)>$	$+\,4<2(81)>$	$+\,2<2(81^2)>$
$+\,4<2(82)>$	$+\,4<2(83)>$	$+\,4<2(84)>$	$+\,2<2(9)>$
$+\,4<2(91)>$	$+\,2<2(91^2)>$	$+\,4<2(92)>$	$+\,4<2(93)>$
$+\,2<2(10\ )>$	$+\,4<2(10\ 1)>$	$+\,2<2(10\ 1^2)>$	$+\,4<2(10\ 2)>$
$+\,2<2(11\ )>$	$+\,4<2(11\ 1)>$	$+\,2<2(12\ )>$	

Now remove all partitions of length greater than 2, replace $\omega_\mu = 2$ by 0, rename the group as $U(2)$ and standardise the $U(2)$ irreps to give the set of $U(2)$ irreps as

$\{0\}$	$+\ 2\{1\}$	$+\ 3\{1^2\}$	$+\ 2\{2\}$
$+\ 4\{21\}$	$+\ 3\{2^2\}$	$+\ 2\{3\}$	$+\ 4\{31\}$
$+\ 4\{32\}$	$+\ 3\{3^2\}$	$+\ 2\{4\}$	$+\ 4\{41\}$
$+\ 4\{42\}$	$+\ 4\{43\}$	$+\ 3\{4^2\}$	$+\ 2\{5\}$
$+\ 4\{51\}$	$+\ 4\{52\}$	$+\ 4\{53\}$	$+\ 4\{54\}$
$+\ 3\{5^2\}$	$+\ 2\{6\}$	$+\ 4\{61\}$	$+\ 4\{62\}$
$+\ 4\{63\}$	$+\ 4\{64\}$	$+\ 4\{65\}$	$+\ 3\{6^2\}$
$+\ 2\{7\}$	$+\ 4\{71\}$	$+\ 4\{72\}$	$+\ 4\{73\}$
$+\ 4\{74\}$	$+\ 4\{75\}$	$+\ 2\{8\}$	$+\ 4\{81\}$
$+\ 4\{82\}$	$+\ 4\{83\}$	$+\ 4\{84\}$	$+\ 2\{9\}$
$+\ 4\{91\}$	$+\ 4\{92\}$	$+\ 4\{93\}$	$+\ 2\{10\ \}$
$+\ 4\{10\ 1\}$	$+\ 4\{10\ 2\}$	$+\ 2\{11\ \}$	$+\ 4\{11\ 1\}$
$+\ 2\{12\ \}$			

No signed sequences in $Sp(4)$ and hence we need simply to multiply, in $U(2)$, by the terms in the $W-$series to weight 12 to get

Now change the group to $SO^*(6)$ and insert a 2 in front of each partition to get
$H^2 =$

$[2(0)]$	$+\ 2[2(1)]$	$+\ 2[2(1^2)]$	$+\ 3[2(2)]$
$+\ 4[2(21)]$	$+\ 3[2(2^2)]$	$+\ 4[2(3)]$	$+\ 6[2(31)]$
$+\ 6[2(32)]$	$+\ 4[2(3^2)]$	$+\ 5[2(4)]$	$+\ 8[2(41)]$
$+\ 9[2(42)]$	$+\ 8[2(43)]$	$+\ 5[2(4^2)]$	$+\ 6[2(5)]$
$+\ 10[2(51)]$	$+\ 12[2(52)]$	$+\ 12[2(53)]$	$+\ 10[2(54)]$
$+\ 6[2(5^2)]$	$+\ 7[2(6)]$	$+\ 12[2(61)]$	$+\ 15[2(62)]$
$+\ 16[2(63)]$	$+\ 15[2(64)]$	$+\ 12[2(65)]$	$+\ 7[2(6^2)]$
$+\ 8[2(7)]$	$+\ 14[2(71)]$	$+\ 18[2(72)]$	$+\ 20[2(73)]$
$+\ 20[2(74)]$	$+\ 18[2(75)]$	$+\ 9[2(8)]$	$+\ 16[2(81)]$
$+\ 21[2(82)]$	$+\ 24[2(83)]$	$+\ 25[2(84)]$	$+\ 10[2(9)]$
$+\ 18[2(91)]$	$+\ 24[2(92)]$	$+\ 28[2(93)]$	$+\ 11[2(10\ )]$
$+\ 20[2(10\ 1)]$	$+\ 27[2(10\ 2)]$	$+\ 12[2(11\ )]$	$+\ 22[2(11\ 1)]$
$+\ 13[2(12\ )]$			

In a precisely similar manner we have $H^3 =$

$[3(0)]$	$+\ 3[3(1)]$	$+\ 5[3(1^2)]$	$+\ 4[3(1^3)]$
$+\ 6[3(2)]$	$+\ 14[3(21)]$	$+\ 13[3(21^2)]$	$+\ 15[3(2^2)]$
$+\ 18[3(2^21)]$	$+\ 10[3(2^3)]$	$+\ 10[3(3)]$	$+\ 27[3(31)]$
$+\ 27[3(31^2)]$	$+\ 40[3(32)]$	$+\ 52[3(321)]$	$+\ 33[3(32^2)]$
$+\ 35[3(3^2)]$	$+\ 50[3(3^21)]$	$+\ 42[3(3^22)]$	$+\ 20[3(3^3)]$
$+\ 15[3(4)]$	$+\ 44[3(41)]$	$+\ 46[3(41^2)]$	$+\ 75[3(42)]$
$+\ 102[3(421)]$	$+\ 70[3(42^2)]$	$+\ 90[3(43)]$	$+\ 135[3(431)]$
$+\ 122[3(432)]$	$+\ 66[3(43^2)]$	$+\ 70[3(4^2)]$	$+\ 110[3(4^21)]$
$+\ 111[3(4^22)]$	$+\ 80[3(4^23)]$	$+\ 35[3(4^3)]$	$+\ 21[3(5)]$
$+\ 65[3(51)]$	$+\ 70[3(51^2)]$	$+\ 120[3(52)]$	$+\ 168[3(521)]$
$+\ 121[3(52^2)]$	$+\ 165[3(53)]$	$+\ 255[3(531)]$	$+\ 242[3(532)]$
$+\ 141[3(53^2)]$	$+\ 175[3(54)]$	$+\ 284[3(541)]$	$+\ 300[3(542)]$
$+\ 232[3(543)]$	$+\ 126[3(5^2)]$	$+\ 210[3(5^21)]$	$+\ 235[3(5^22)]$
$+\ 28[3(6)]$	$+\ 90[3(61)]$	$+\ 99[3(61^2)]$	$+\ 175[3(62)]$
$+\ 250[3(621)]$	$+\ 186[3(62^2)]$	$+\ 260[3(63)]$	$+\ 410[3(631)]$
$+\ 402[3(632)]$	$+\ 246[3(63^2)]$	$+\ 315[3(64)]$	$+\ 522[3(641)]$
$+\ 570[3(642)]$	$+\ 308[3(65)]$	$+\ 525[3(651)]$	$+\ 210[3(6^2)]$
$+\ 36[3(7)]$	$+\ 119[3(71)]$	$+\ 133[3(71^2)]$	$+\ 240[3(72)]$
$+\ 348[3(721)]$	$+\ 265[3(72^2)]$	$+\ 375[3(73)]$	$+\ 600[3(731)]$
$+\ 602[3(732)]$	$+\ 490[3(74)]$	$+\ 824[3(741)]$	$+\ 546[3(75)]$
$+\ 45[3(8)]$	$+\ 152[3(81)]$	$+\ 172[3(81^2)]$	$+\ 315[3(82)]$
$+\ 462[3(821)]$	$+\ 358[3(82^2)]$	$+\ 510[3(83)]$	$+\ 825[3(831)]$
$+\ 700[3(84)]$	$+\ 55[3(9)]$	$+\ 189[3(91)]$	$+\ 216[3(91^2)]$
$+\ 400[3(92)]$	$+\ 592[3(921)]$	$+\ 665[3(93)]$	$+\ 66[3(10\ )]$
$+\ 230[3(10\ 1)]$	$+\ 263[3(10\ 1^2)]$	$+\ 495[3(10\ 2)]$	$+\ 78[3(11\ )]$
$+\ 275[3(11\ 1)]$	$+\ 91[3(12\ )]$		

as found previously by more tedious methods.

Likewise to determine $H \otimes \{2\}$ for $SO^*(6)$ one first computes
$(S \otimes \{2\})_2 =$

$< 2(0) >$	$+ < 2(1) >$	$+ < 2(1^2) >$	$+ < 2(2) >$
$+ 2 < 2(21) >$	$+ 2 < 2(2^2) >$	$+ < 2(3) >$	$+ 2 < 2(31) >$
$+ 2 < 2(32) >$	$+ < 2(3^2) >$	$+ < 2(4) >$	$+ 2 < 2(41) >$
$+ 2 < 2(42) >$	$+ 2 < 2(43) >$	$+ 2 < 2(4^2) >$	$+ < 2(5) >$
$+ 2 < 2(51) >$	$+ 2 < 2(52) >$	$+ 2 < 2(53) >$	$+ 2 < 2(54) >$
$+ < 2(5^2) >$	$+ < 2(6) >$	$+ 2 < 2(61) >$	$+ 2 < 2(62) >$
$+ 2 < 2(63) >$	$+ 2 < 2(64) >$	$+ 2 < 2(65) >$	$+ 2 < 2(6^2) >$
$+ < 2(7) >$	$+ 2 < 2(71) >$	$+ 2 < 2(72) >$	$+ 2 < 2(73) >$
$+ 2 < 2(74) >$	$+ 2 < 2(75) >$	$+ < 2(8) >$	$+ 2 < 2(81) >$
$+ 2 < 2(82) >$	$+ 2 < 2(83) >$	$+ 2 < 2(84) >$	$+ < 2(9) >$
$+ 2 < 2(91) >$	$+ 2 < 2(92) >$	$+ 2 < 2(93) >$	$+ < 2(10\ ) >$
$+ 2 < 2(10\ 1) >$	$+ 2 < 2(10\ 2) >$	$+ < 2(11\ ) >$	$+ 2 < 2(11\ 1) >$
$+ < 2(12\ ) >$			

and continues as before to finally obtain
$H \otimes \{2\} =$

$[2(0)]$	$+ [2(1)]$	$+ 2[2(2)]$	$+ 2[2(21)]$
$+ 3[2(2^2)]$	$+ 2[2(3)]$	$+ 2[2(31)]$	$+ 3[2(32)]$
$+ 3[2(4)]$	$+ 4[2(41)]$	$+ 6[2(42)]$	$+ 4[2(43)]$
$+ 5[2(4^2)]$	$+ 3[2(5)]$	$+ 4[2(51)]$	$+ 6[2(52)]$
$+ 4[2(53)]$	$+ 5[2(54)]$	$+ 4[2(6)]$	$+ 6[2(61)]$
$+ 9[2(62)]$	$+ 8[2(63)]$	$+ 10[2(64)]$	$+ 6[2(65)]$
$+ 7[2(6^2)]$	$+ 4[2(7)]$	$+ 6[2(71)]$	$+ 9[2(72)]$
$+ 8[2(73)]$	$+ 10[2(74)]$	$+ 6[2(75)]$	$+ 5[2(8)]$
$+ 8[2(81)]$	$+ 12[2(82)]$	$+ 12[2(83)]$	$+ 15[2(84)]$
$+ 5[2(9)]$	$+ 8[2(91)]$	$+ 12[2(92)]$	$+ 12[2(93)]$
$+ 6[2(10\ )]$	$+ 10[2(10\ 1)]$	$+ 15[2(10\ 2)]$	$+ 6[2(11\ )]$
$+ 10[2(11\ 1)]$	$+ 7[2(12\ )]$		

again in agreement with previous more tedious methods.