Physics as a Journey

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Once you enter the world of science or mathematics or philosophy, endless plains open around you. The more you learn, the more fascinating the whole thing becomes
— Colin Wilson, *Voyage to a Beginning*, 1968

Preface

These lecture notes started at the University of Canterbury, New Zealand as part of a course entitled *The Physical Universe*. The original intent had been to develop a course that would give a broad picture of the physicists conception of the Universe. Initially it represented an attempt to interest a wide range of students, outside of physics, in the whole sweep of physics and create a greater appreciation of physics in the community. At the first presentation the audience included many students from the Arts, Law and Commerce departments as well as non-physics science students, all of whom could gain credit for the course. The emphasis was upon concepts rather than mathematical detail. Physics students demanded to be included in the course as they felt the course brought a degree of coherence and overview to their subject not obtainable in their more specialised courses. The University recognised the value of the course to both non-science and science students and extended credit to all participants.

Since coming to Poland I thought that such a course might appeal to students and developed the current course *Physics as a Journey* to be given in English to students of physics. This served the dual purpose of giving students a broad overview of physics and the opportunity to listen to lectures in English. The choice of the title *Physics as a Journey* reflects my interest in viewing physics as a historical journey and also my close interest in the relationship between science and technology - neither can survive on its own. Science advances as technology advances and vice versa. The discovery of the electron could not have preceded the development of vacuum technology. Likewise the performance of the Michelson-Morley experiment at the time of Copernicus was fortunately not possible!

Lecture notes omit the spark of delivery, the numerous side comments and the demonstrations which breath life into the course from which they are taken. Nor do they reflect the participatory audience. These notes are no exception - they represent the course in its broadest outline. I hesitate from the onerous task of weaving these notes into a more coherent presentation. I hope at least the reader will gain some appreciation of the joys and concepts of physics and perhaps consider delving more into the subject matter of these notes.

B. G. Wybourne

Technika dała duszy wszechpotęgę. Ale i przygniotła ją. Pojawiła się ,,techniczna dusza", posiadająca mechanizm twórczy, lecz pozbawiona twórczego natchnienia.
— Wasilij Rozanow, *Aforyzmy*

Chapter Headings

Chapter One

The Journey Begins

1. Introduction

Physics enters all most every aspect of modern life. Sophisticated technology now dominates our lives for both better and worse. It has not always been this way. The technological revolution is largely a product of this century, indeed even more so of the latter half of this century. Physics in all its diversity has played a major, probably the major, role in fomenting this revolution. There has been a close interplay between science and technology that has often been overlooked or misunderstood. Science has fed technology and technology has fed science. The two coexist and yet are distinguishable.

Physics has not only led to advanced technology but has also fundamentally changed our perspective of the universe in which we live. Physics at the close of the twentieth century is vastly different in its view of the world than at the beginning of this century. Based on our past experience it would be hazardous to predict the developments in both science and technology that will occur in the imminent twenty-first century.

These notes reflect my conviction that it is important to understand the origins of the dual revolution in physics and technology, to know something of the long journey that has lead us to the present, and to see how and why our concepts of the universe have changed. This is, in my opinion, too important to be left to just the physicists but must be communicated to a larger world of people of diverse cultures, educational experience, and occupation. The central problem is one of communication to people of such differing backgrounds. Even physicists themselves often fail to see the inherent unity of their discipline and to grasp the bigger picture or to know the historical orgins of their subject.

The physicist is most at home using the language of mathematics, with its own deep concepts. This in itself makes communication difficult. This also leads to the popular misconception that

$$\boxed{\boxed{physics = equations}}$$

Some people think that physics is over once the equation is found which governs some phenomena. To me this seems as foolish as somebody who says English is over once he has learned the words and the grammar and never goes on to read and understand Shakespeare. Physics is not the equation but the multitude of phenomena which result from it. To know the equation is not the end but the beginning and to deduce from it the physics is an unending quest. (W. E. Thirring 1987)

My objective is to give a broad sweep of our subject, largely free of mathematical equations and not demanding a previous deep knowledge of physics. This makes the task of communicating more difficult but as Rutherford noted in a pre-feminist age "*If you can't explain your ideas to the charwoman then you don't understand them*". I would hope that much of what I discuss will also be of significance to experienced physicists whom in their busy life have often missed out on the historical development of their subject and on the often peculiar happenings that have lead to real advances for the wrong reasons.

In this opening chapter I propose to first give a broad sweep of the key developments leading up to the present time and then to return to earlier times. Finally we shall look to the limitations of classical physics. Many of our discussions will be illustrated by simple experiments and most chapters conclude with a set of questions to stimulate further thought.

2. The On-Going Physics Revolution

To fully understand the origins of the physics revolution one needs a perspective of the key developments that occurred in the past. Any such list is incomplete and ignores many small developments that preceded major syntheses and new conceptual insights. Great names survive but lesser names often made the great names possible. Here I only indicate some of the highlights. Later we shall explore these in more detail.

I. MECHANICS

Ptolemy (100-168)

Develops an early geocentric model of the solar system with the Earth being seen as the centre of the Universe. Used for the next fourteen centuries for navigation. Discrepancies with observation were minimised by introducing epicycles. The Ptolemaic model was in broad agreement with the prevailing intuition of the sun rising and setting, the stars appearing to revolve about the earth and the apparent immovablity of the Earth.

Copernicus (1473-1543)

Develops a heliocentric model of the solar system. The planets are considered to revolve about the sun in circular orbits. Copernicus has shifted the origin away from the Earth to the Sun. Still there is little theoretical basis to the model and for purposes of navigation appears to be inferior to tables prepared on using the Ptolemaic model. The Copernicus picture gives an alternative, and sometimes more elegant interpretation of phenomenon such as the rising and setting of the sun. Epicycles are still used for details of planetary motion. Copernicus's theory meets with strong opposition, his theory is counter intuitive. It is easy for we of the twentieth century to ridicule the opposition faced by Copernicus. However, hindsight ignores the culture prevailing at the time and the observational experience of the time. Indeed it is likely that the majority of those living in the twentieth century could still not adequately answer the questions put to Copernicus. Copernicus also enlarged the concept of the scale of the Universe and started to develop some idea of the vastness of the Universe.

Galileo (1564-1642)

Galileo investigates the motion of bodies both experimentally and theoretically. The science of mechanics starts to develop particularly in his introduction of the concept of inertia and his recognition of the relativity of mechanics. This is the start of the quantification of the laws of physics.

Newton (1642-1727)

Newton introduces his law of gravitation. Assumes his law has universal application and can be applied to celestial objects. This makes it possible to predict positions of celestial objects and establishes celestial mechanics. Detailed calculations of planetary and comet orbits become possible.

II. ELECTROMAGNETISM

Faraday (1791-1867)

Entirely without any mathematics background Faraday establishes the experimental study of electric and magnetic phenomenon. His collected works encompass three volumes devoid of any equations and are yet rich in insights into the nature of electric and magnetic fields. Indeed the field concept derives from Faraday. His experimental work was to form the basis of later developments in electromagnetic theory.

Maxwell (1831-1879)

Maxwell does to electricity and magnetism what Newton did to mechanics. He produces a theoretical synthesis of electricity and magnetism to produce a single unified theory known as electromagnetism. It then becomes possible to calculate electromagnetic effects and thus to quantify the subject. Electrical engineering becomes possible. Further, Maxwell establishes light as an electromagnetic wave. Maxwell's collected works are also in three volumes with no pages devoid of equations.

III. RELATIVITY AND QUANTUM THEORY

Planck (1858-1947)

Planck, steeped in classical physics and more particularly thermodynamics, is forced to introduce his quantisation postulate and establishes his black-body radiation law. This was to lead to the development of modern quantum theory, of enormous technological significance.

Einstein (1879-1955)

Einstein views light as a quantum phenomena and convinces the physicists of the significance of the quantum theory. In parallel he develops the theory of relativity and completes Maxwell's unification of electricity and magnetism. Einstein changes our concepts of space, time, energy and mass. Newtonian mechanics becomes an approximation of relativistic mechanics.

de Broglie (1892-1987)

de Broglie quantises particles attributing to them wave-like properties.

Dirac (1902-84)

Dirac combines quantum theory with relativity producing his relativistic wave equation. The concept of matter and antimatter follows.

Feynman, Schwinger and Tomonaga

These three physicists played a key role in quantising electromagnetism to produce quantum electrodynamics (QED).

IV. PARTICLE PHYSICS AND COSMOLOGY

Gell-Mann (1929-)

Gell-Mann's introduction of the quark model of particles such as neutrons and protons represents the start of a great synthesis of the diversity of particles produced by experimentalists.

Penzias and Wilson

The 1964 observation by Penzias and Wilson of the cosmic relic background black-body radiation had a major role in the acceptance of the Big-Bang model as the standard model of the Universe.

Glashow, Salam & Weinberg

In 1967 Glashow, Salam and Weinberg successfully unified the weak force of radioactivity with the electromagnetic force to produce the unified electro-weak theory. The predicted $W^{\pm}$, Z^0 particles were discovered in 1984.

Green & Schwarz

In 1984 Green & Schwarz developed a string model and thence a superstring model in an attempt to complete the unification of the strong force that acts in the nucleus with the electro-weak force and the gravitational force. The dream of unifying all the forces into a single **T**heory **O**f **E**verything (TOE) remains just that but recent developments suggest that the attainment of the dream may be possible.

3. Do Heavy Bodies Fall Faster than Lighter Ones?

The answer to the above question was given by Galileo Galilei in his book *Dialogue Concerning the Two Chief World Systems - Ptolemaic & Copernican.*

To the followers of Aristotle (~400BC) it was intuitively obvious and in accord with some observations that heavy bodies fell faster than lighter ones. This idea was challenged by Galileo Galilei (~1628). Galileo argued that in a vacuum all bodies would fall at the *same* rate. Aristotle had assumed that heavy falling bodies move in proportion to their weight .

Consider a heavy stone connected to a light one by a string. According to Aristotle, when released, the heavy one pulls the lighter one down and tries to make it fall faster than it would if it was unattached. The light one, on the other hand, tends to slow down the heavier one. But the combined body is heavier than the heavy one. It should therefore fall faster than the heavy one! The Aristotelian view thus leads to a contradiction if we ignore air resistance. Galileo concluded that *all bodies fall at the same rate in a vacuum.* This was an early example of a 'gedanken' or 'thought' experiment.

Note that Galileo tries to reduce the problem to its simplest form - in vacuum. In one stroke he removes all extraneous and detracting influences, resistance of air, winds etc. Only after studying the simplest case does one normally move on to more complicated effects. If we cannot understand the simplest situation there is little point in considering more complex problems. The equality of the rate of fall of a feather and a stone was demonstrated at the site of the first lunar landing. If we drop a large and small coin in a cylinder of water we observe the heavier coin to sink faster than the smaller coin.

Why?

4. Mechanics

Mechanics is associated with the reponse of material objects to applied forces and divides into two major areas:

1. **Statics** where the objects of interest are stationary or at rest *relative* to the observer. e.g. a ladder leaning on a wall.
2. **Dynamics** where the objects are in motion with respect to a given observer. In a sense statics is a limiting case of dynamics.

Classical mechanics arose from the early studies of Galileo and Newton. Under Newton laws of motion were established that led to the description of the dynamics of bodies moving under the influence of the force of gravity. Major developments came from Laplace, Lagrange, Hamilton and Jacobi. In the Hamilton-Jacobi formulation of classic dynamics the objective was to express the time evolution of a given system in terms of initial conditions and thus to be able to predict the future state of a system of objects from a knowledge of its state at an earlier time - i.e. the time evolution of the system.

5. Laplace's Dictum

Given the initial positions and velocities of every particle in the universe it should be possible to predict every future event in the universe

NB This implies a totally deterministic (clockwork) universe.

Is Laplace's statement a valid point of view? Is classical mechanics totally deterministic? Let us consider the case of uniform motion.

6. Uniform Motion

Suppose an object moves with a uniform constant speed of 10 metres/second. After 10 seconds it will have covered a distance of 10 x 10 = 100 metres. Suppose the object starts initially at a time $t = 0$ at a distance x_0 to the right of a marker at a speed of v/metres/second to the right.

After two seconds it will be at a distance

$$x = x_0 + 2v$$

metres from the marker. After t seconds

$$\boxed{\boxed{x = x_0 + vt}} \tag{1}$$

Eq.(1) is an example of a *mathematical relation.* It allows us to *predict* the position of the object at any future time t knowing the *initial values* of the position x_0 and the speed v_0 and the time t_0. Eq. (1) suggests that in the case of uniform motion we have complete predictability in terms of the initial values of the position and velocity.

7. Is Classical Mechanics Deterministic?

Let us consider a perfectly elastic ball C moving between two perfectly rigid walls A and B. Can we predict the position of the ball at all future times?

Since C is perfectly elastic and the walls are perfectly rigid the ball will bounce off either wall with no change in its *speed* v. Suppose A and B are 1 metre apart and the ball slides on a perfectly frictionless floor between A and B with and initial speed of v = 1metre/second starting at wall A. Can we predict the position of C at any future time?

After	1 sec	B
	2 sec	A
	2.5 sec	midway
	982 sec	A

It would appear we have achieved complete predictability. However, we have assumed we know v with *infinite precision.* What if v was somewhere between 0.7 and 1.3 metres/second?

Let us determine the position of the ball from wall A at one second intervals for assumed values of v between 0.7 and 1.3 metres/second.

v	$t = 1$	2	3	4
0.7	0.7	1.4	2.1	2.8
0.8	0.8	1.6	2.4	3.2
0.9	0.9	1.8	2.7	3.6
1.0	1.0	2.0	3.0	4.0
1.1	1.1	2.2	3.3	4.4
1.2	1.2	2.4	3.6	4.8
1.3	1.3	2.6	3.9	5.2

At $t = 1$ second the ball has travelled somewhere between 0.7 and 1.3 metres from A. At $t = 2$ seconds the ball has travelled somewhere between 1.4 and 2.6 metres while at 4 seconds it is somewhere between 2.8 and 5.2 metres.

Thus if the initial speed of the ball is only known to within 1 ± 0.3metres/second then after 4 seconds we can no longer predict where the ball is between the walls. What if we improve our accuracy of measurement of v ? If we determine the initial speed to be $1 \pm \epsilon$ metres/second then after t seconds the ball will have travelled a distance of $t \pm \epsilon t$ metres. Thus if we attained an accuracy of $v = 1 \pm 10^{-4}$ metres/second then after 10^4 seconds we could not say where the ball is other than somewhere between the walls. *Regardless of the precision with which we determine the initial position and speed of the ball after a finite time we will cease to be able to make a meaningful prediction of the position of the ball between the walls.*

SIDE REMARK

From the equation

$$x = x_0 + vt$$

if x_0 is between x_0 and $x_0 \pm \delta x$ and v is between v and $v \pm \delta v$ then

$$\begin{aligned} x &= (x_0 \pm \delta x) + (v \pm \delta v)t \\ &= (x_0 + vt) \pm \delta x \pm \delta vt \end{aligned}$$

The error in the initial position is constant in time whereas the error in the initial speed produces an error in the position that grows with the time t.

8. Can we ignore External Bodies

E. Borel 'Introduction geómétrique á la physique" (Gauthier-Villars, Paris, 1912) has noted that the motion of one gram of matter by $1cm$ on a not too distant star (say, Sirius) would make a change of about 10^{-100} in the gravitational field on the earth leading to a change of 10^{-100} in the initial positions and velocities of the molecules of a gas making it impossible to compute the motion of these molecules for more than 10^{-6} seconds.

Clearly to make any progress we have to make approximations and these will prevent us from making precise predictions of the time evolution of a system. Improving the precision of measurement only postpones the time when we lose the ability to make a meaningful prediction. We need to remember that in physics we always deal with model systems. Real systems are extremely complex and are approached by developing more sophisticated models involving fewer approximations.

9. Trajectories in Classical Physics

> *It is impossible to study the properties of a single mathematical trajectory. The physicist knows only bundles of trajectories, corresponding to slightly different initial conditions*
> — Leon Brillioun

Classical mechanics as developed by Newton, Lagrange, Euler, Laplace, Jacobi, Hamilton and others was concerned with the prediction of the time evolution of physical systems in terms of given *initial conditions*. Thus the *trajectories* or *paths* of particles were expressed in terms of the initial conditions. Different initial conditions lead to different trajectories. If the initial conditions differ slightly then the trajectories will initially be close together but as time passes the trajectories will become increasingly divergent.

The prediction of the time evolution of a classical system depends on the accuracy with which the initial conditions can be determined. The concept of trajectories is crucial to classical mechanics. At the subatomic level the classical picture of a deterministic trajectory is lost. The generation of unpredictable outputs from a deterministic system is part of the modern subject of chaos.

10. Trajectory of a Projectile

The trajectory of a projectile in the earth's gravitational field depends on its initial speed v and its initial angle θ of elevation. Each choice of (v, θ) leads to a different trajectory. Galileo showed that the trajectory of such a projectile follows a parabola. Given an initial and final position there is an infinity of trajectories that will carry a projectile from the initial to the final position each involving a different set of initial speed and elevation.

11. Limitations of Classical Mechanics

1. Assumes structureless particles.
2. Assumes initial conditions can be determined precisely.
3. Assumes a completely deterministic universe.

12. Statistical Mechanics

For systems involving large numbers of particles one can develop statistical models such as those of Boltzmann where one tries to predict the bulk behaviour of a large ensemble while abandoning a detailed knowledge of the dynamics of individual particles.

13. Physical Systems and Models

Physicists proceed by considering ideal systems that can be isolated from the rest of the universe. Effects due to objects outside the system are assumed to be negligible or capable of being averaged out. Any such systems represent an abstraction of reality. Any system that is less than the universe itself must involve initial conditions that are of finite precision.

14. Questions

Q1. Give other examples where predictive ability is lost with the passage of time.

Q2. Copernicus's contempories argued against his hypothesis that the earth moved and in particular spun on its axis by saying that if they jump up in the air they land on the same spot from which they jumped whereas if Copernicus's earth rotates they would consider jumping a very dangerous practice. Furthermore birds flying to the East flew neither faster nor slower than birds flying to the West. How would YOU answer Copernicus's critics?

Q3. To improve long range weather forecasting would it be better to get much larger and faster computers or to increase the accuracy of wind and temperature measurements at more sites?

Q4. A golfer attempts to make a hole-in-one at a distance of 100metres with a hole 10cm in diameter. What are the essential initial conditions that must be satisfied? What other factors might affect the outcome? Does a hole-in-one require good luck or exceptional skill or both?

Q5. A good athlete can cover 10km in under 13minutes in 25 laps on a 400metre track. Currently time measurements are quoted to 0.01seconds. Is this accuracy sensible?

Q6. Why did dinosaurs have small heads while whales have large heads?

Every effort has been taken to present the mathematical developments in this chapter in a comprehensible logical sequence. But the nature of the developments simply does not allow a presentation that can be followed in detail with modest effort: the reductions that are necessary to go from one step to another are often very elaborate and, on occasion, may require as many as ten, twenty, or even fifty pages. In the event that some reader may wish to undertake a careful scrutiny of the entire development, the author's derivations (in some 600 legal-size pages and in six additional notebooks) have been deposited in the Joseph Regenstein Library of the University of Chicago.
— S. Chandrasekhar, *The Mathematical Theory of Black Holes*

If it be true that the impetus with which the ship moves remains indelibly impressed in the stone after it is let fall from the mast; and if it be further true that this motion brings to impediment or retardment to the motion directly downwards natural to the stone, then there ought to ensue an effect of a very wondrous nature. Suppose a ship stands still, and the time of the falling of a stone from the mast's roundtop to the deck is two beats of the pulse. Then afterwards have the ship under sail and let the same stone depart from the same place. According to what has been premised, it shall take the time of two pulses in its fall, in which time the ship will have gone, say twenty yards. The true motion of the stone will then be a transverse line (i.e., a curved line in the vertical plane), considerably longer than the first straight and perpendicular line, the height of the mast, and yet nevertheless the stone will have passed it in the same time. Increase the ship's velocity as much as you will, the falling stone shall describe its transverse lines still longer and longer and yet shall pass them all in those selfsame pulses.
— Sagredo in Galileo's *Two New Sciences*

Chapter Two

Of Scale and Motion

Our question is, why snowflakes in their first falling before they are entangled in larger plumes, always fall with six corners and with six rods, tufted like feathers. ... There must be some definite cause why, whenever snow begins to fall its initial formations invariably display the shape of the six-cornered starlet
— J. Kepler *The Six-Cornered Snowflake, (1611)*

1. SYNOPSIS We first discuss the concept of scale which has important implications in physics, biology and industry. We then develop Galileo's concept of Inertia and establish units or measures for force and energy. We briefly discuss Newton's Laws of Motion and Einsteins Mass-Energy Equivalence and its application to space exploration.

2. Changes of Scale

The concept of scale, and of changes of scale, plays an important role in physics and failure to appreciate these concepts can lead to catastrophes. It has been known since Greek times, at least, that the area, $A_\circ$, enclosed by a circle of radius r is given by

$$\boxed{A_\circ = \pi r^2} \tag{1}$$

while the area, A_{sphere} was found to be

$$\boxed{A_{sphere} = 4\pi r^2} \tag{2}$$

The volume, V_{sphere}, of a sphere of radius r was known to be

$$\boxed{V_{sphere} = \frac{4}{3}\pi r^3} \tag{3}$$

From these three elementary results can follow profound conclusions.

3. Why do small animals have a higher rate of metabolism than big animals?

Imagine we have a small sphere and a large sphere both at a temperature T. The total heat content of a sphere will proportional to the *cube* of its radius while the heat radiated from the sphere will be proportional to its surface area and hence to the *square* of its radius. Thus

$$\boxed{\frac{\text{Heat lost by sphere}}{\text{Heat content of sphere}} \propto \frac{1}{r}} \tag{4}$$

Thus the ratio depends on the inverse of the radius of the sphere. The larger the sphere the smaller is the ratio. This leads us to expect that small animals will lose heat, in relationship to their size, faster than large animals and hence will need to metabolise food more rapidly than large anuimals as observed. By the same reasoning we expect babies to be more susceptible to temperature changes than adults which is why they are provided with better insulating clothes than adults.

4. Why do Dinosaurs have small heads?

Let us model a small dinosaur by small sphere (the head) connected by a cylindrical rod (the neck) to a larger sphere (the body). What happens if the dinosaur grows and each characteristic radius is simply scaled? The head and body would grow as the *cube* of the radii but the *strength* of the neck will be proportional to its cross-sectional area *consider muscles* and hence as the *square* of its radius. Thus if the head grows in proportion to the body of the dinosaur it will rapidly outgrow the strength of its neck. Such a characteristic can be seen in most animals. The head of the infant relative to its body is significantly larger than for the adult animal.

It was Galileo who first pointed out that scaling lead to limits of the size of animals. In the case of whales the head scales and there is no neck. There the bouyancy of the water overcomes the force of gravity experienced by the land dwelling animals and of course whales become helpless out of water.

5. Lessons from Scaling

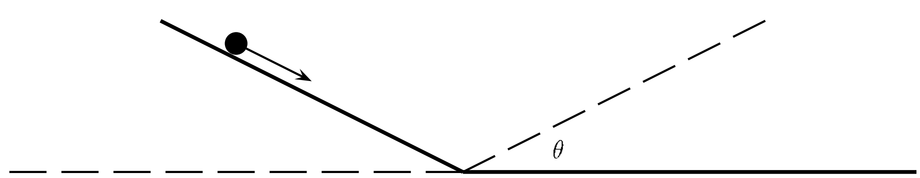
Failure to appreciate the significance of scaling effects has been the source of repeated industrial problems and failures. A pilot plant is designed and found to work and then it has been simply scaled for industrial production and often found not to work. When we change the scale of objects often new properties arise that are not noted on the small scale.

Gravitational forces are extremely weak, indeed the weakest of all known forces. In describing the properties of a small object they can be wholly neglected but for large objects such as the sun or in super novae they can become overwhelming.

A cubic cm of ^{239}Pu weighs about 19grams and can be safely carried in the pocket if enclosed in a plastic bag. A 400 cubic centimeter sphere, of diameter about 9cm becomes a fearsome object.

6. Galileo's Law of Inertia

Galileo examined the motion of a ball rolling down an inclined plane.



He observed that a ball going down an inclined plane would start from rest, accelerate to the bottom and then rise up an inclined to the same height as it started, momentarily coming to rest. As the angle θ became smaller the ball travelled further. Galileo argued that if the second plane was horizontal then in the absence of friction the ball would continue in motion along a horizontal line at the speed it had at the bottom of the first plane. This observation led to Galileo's *law of inertia.* The inertia of a body is the property of a body that tends to resist change in its state of rest or motion.

Galileo formulated his law of inertia as

> *A body will remain at rest or continue to move with a constant speed in a straight line unless acted upon by an outside agent.*

7. Consequences of Galileo's Law of Inertia

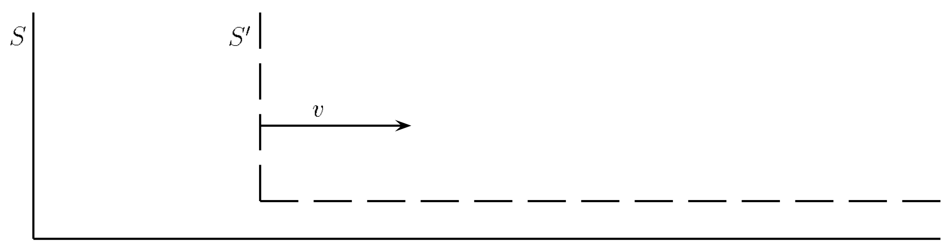
Galileo used his law of inertia to demolish the mechanics of Aristotle and to establish the Copernican revolution. Following Galileo, imagine a ship travelling at a constant speed v *relative to* the earth. Imagine a stone dropped from the crows-nest of the ship. If the ship was at rest everyone expects the stone to fall straight down to the bottom of the mast. Galileo claimed that if the ship was moving at a constant speed relative to the earth the stone would still land at the foot of the mast

Galileo reasoned that before the stone is released it is travelling along as part of the ship. When the stone is released the stone's inertia keeps it moving with the same speed along the horizontal straight line. The ship and the stone continue to move together horizontally but gravity pulls the stone vertically downwards. Galileo conjectured that the vertical motion does not interfere with its horizontal motion. So as the stone falls, it continues to move horizontally with constant speed dropping to the bottom of the mast. A sailor on the ship would see the stone fall in the same way on a moving ship as one at rest relative to the earth. Seen by an observer on the earth rather than the ship the stone will be seen to fall with a parabolic path.

9. Relative Motion and Frames of Reference

Imagine you are the only object in the universe. Are you at rest or in motion? Does the question make sense? No! you have no frame of reference. Now imagine there are two objects in the universe. You can now discuss the *relative motion* of one object with respect to the other. NB. you can only speak of the *relative* motion, you can not say one is moving while the other is at rest. All motion is relative - there is no such thing as *absolute rest*.

A reference frame where Galileo's law of inertia holds is known as an *inertial frame of reference*. Any other reference frame that is moving at a constant speed in a straight line with respect to a given inertial frame of reference is also an inertial frame of reference.



Thus S and S' could be two inertial frames of reference. We can say S' is moving with a speed v *relative* to S. We can make no statement as to the absolute motion of S or S'.

N.B. An inertial frame of reference is a LOCAL frame of reference, NOT a GLOBAL frame of reference.

10. Galilean Relativity

Galileo formulated his *relativity principle* as

It is impossible for an observer in an inertial frame of reference to detect any motion by any experiment performed entirely within that reference frame.

Let us now briefly review some elementary concepts and establish our units of measurement for the rest of the course.

11. Velocity and Speed

We define the *speed* of an object relative to an inertial frame as the rate of of change of the position of the object with respect to time. The average speed is just the distance travelled divided by the time taken. The instantaneous speed s is then

$$\boxed{\boxed{s = \frac{dx}{dt}}} \tag{5}$$

The concept of the *velocity* of an object includes not only its speed but also the *direction* of its motion in a given frame of reference and is a *vector quantity*. Thus the instantaneous velocity $\mathbf{v}$ is

$$\boxed{\boxed{\mathbf{v} = \frac{d\mathbf{x}}{dt}}} \tag{6}$$

12. Acceleration

The concept of acceleration, unknown to Aristotle, was developed by Galileo. An object is said to be accelerating if its velocity is changing with time and like velocity is a vector quantity. Thus the instantaneous acceleration **a** is

$$\boxed{\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{x}}{dt^2}} \tag{7}$$

A body falling freely near the earth's surface experiences an acceleration due to the earth's gravity of $10ms^{-2}$ directed towards the earth's centre.

13. Mass and Inertia

Mass may be regarded as a quantitative measure of inertia and will be measured in *kilograms*

14. Newton's Laws of Motion

Mechanics became a truly quantitative subject with Newton's enunciation of three laws of motion.

1. *A body at rest or in uniform motion will remain at rest or in uniform motion unless some external force is applied to it.*
2. *When a body is acted upon by a constant force* ($\mathbf{F}$) *its resulting acceleration* ($\mathbf{a}$) *is proportional to the force and inversely proportional to its mass* (m)*. i.e.*

$$\boxed{\mathbf{F} = m\mathbf{a}} \tag{8}$$

3. *To every action there is an equal and opposite reaction.*

The first law was certainly known to Galileo. Eq.(8.) is one of the most important equations in physics though less well-known to lay persons than Einstein's energy-mass equivalence equation. Note *force* is a vector quantity.

15. Newton's Universal Law of Gravity

Newton identified the force of gravity as a universal force acting between bodies. It has an infinite range r of action, occurs only as an attractive interaction, and the magnitude of the force F is directly related to the product of the two interacting masses (m_1, m_2) decreasing with the square of the distance separating their centres-of-gravity. Thus

$$\boxed{F_G = \frac{Gm_1m_2}{r^2}} \tag{9}$$

This was the first example of a universal force equation. The universal gravitational constant.

$$\boxed{G = 6.67 \times 10^{-11} N.m^2.kg^{-2}}$$

is the force between two one kilogram masses separated by one metre.

16. Einstein's Mass-Energy Relationship

We shall deal with this subject more fully in the second semester but we shall need the result this semester. One of the highlights of Einstein's theory of relativity was his identification of mass-energy equivalence via his celebrated result

$$\boxed{E = mc^2} \tag{10}$$

where c is the speed of light and m is the mass. This result tells us that matter and energy are interconvertible. A 1kg mass will have an energy equivalence of

$$\begin{aligned} E &= (3 \times 10^8)^2 \\ &= 9 \times 10^{16} \quad \text{Joules} \\ &= 2.5 \times 10^{10} \quad \text{kWh} \end{aligned}$$

which is enough energy for every person in the world to run a one bar heater for six hours. In the case of the sun about 4 million tonnes of matter are converted into energy every second.

17. Relativity, Space Travel and Star Trek

Consider a one way journey to a galaxy 10^9 light years distance from us. We might conclude that if we travelled at the speed of light we would age 10^9 years by the time we reach our destination. However, Einstein's theory of relativity assures us that such is not the case. The traveller measures a shorter time interval since the traveller's clock runs slower than an earth based clock.

Suppose the traveller's aging for the journey is to be 10 years. Einstein's relativity assures us that if the rocket travels very close to the speed of light (to within $10^{-8} ms^{-1}$) the traveller need only age 10 years. This seems to simply solve the deep space travel problem. Sci-Fi looks like becoming a reality!

WAIT!

HOW MUCH ENERGY IS REQUIRED TO LAUNCH A ONE KILOGRAM OBJECT TO REACH THE DESIRED SPEED?

Einstein's relativity gives us the answer : To accelerate 1kg so as to reach the desired speed is at least

$$\boxed{3 \times 10^{18}\mathrm{J}}$$

How much energy is that? In the USA the average person uses 10^5kWh/year. Assume a world population of 4×10^9 persons all at the USA standard of energy consumption

$$\boxed{\text{i.e. } 4 \times 10^{14} \text{ kWh/year}}$$

Thus the entire world's energy production would be required for nearly 10^4years to accelerate just 1kg.

18. The Perils of Interstellar Matter

Even if we could make such a space ship we would need to add to our space ship massive shielding to protect the crew from radiation produce by collisions of the ship with the sparsely distributed hydrogen atoms of interstellar matter. Seen from the space ship this would result in radiation equivalent to that produced by the largest particle accelerators on earth. The perils of interstellar matter are largely overlooked by Sci Fi.

19. Questions

Q1. Why are there only large animals (e.g bears and seals) and large birds (e.g. penguins) in the polar regions?

Q2. An Aristotlelian claims there is no such thing as inertia because when a stone is released it does not remain in mid-air but falls, whereas the law of inertia states that it should remain at rest. How do YOU reply to this criticism?

Q3. If the earth spins on its axis how come birds can fly East to West or West to East with equal ease?

Q4. You are travelling in a super-silent plane that is travelling with a constant speed relative to the earth. The curtains are drawn for the movie. Can you tell that you are moving?

5. You are travelling on a ship in a calm sea in a straight line at a uniform speed relative to the earth. You play a game of billiards. Will your game be any different from that in a billiard saloon on earth? An observer is hovering over the earth in a balloon that is stationary with respect to the earth. The observer looks down on the billiard table on the ship. Describe in words the trajectories seen by the observer.

Q6. Your spaceship is moving in deep space at a constant speed relative to a distant star. A mechanical arm reaches into the garbage bay, pulls out a load of garbage and releases it. Describe the motion of the garbage from the point of view of a passenger on the space ship.

Q7. A plane drops a bale of hay while in level flight to cattle down below. The plane moves at a constant speed relative to the earth. (a) What is the path the pilot sees for the falling bale? (2) What is the path seen by the farmer down below?

Q8. You are travelling in a car at a constant speed relative to the road and the car turns a bend . You are "thrown" to one side. (a) Which side? (b) Why?

Q9. Devise a gadget to test whether you are in an inertial frame or not.

Q10. An object travels in a circle at a constant speed of sms^{-1}. Describe the objects velocity. Is the object's velocity constant?

Q11. A particle travels at a constant speed s in a circle. What is the particle's acceleration and in what direction?

Q12. Would the Earth be an interesting place to visit at the end of an interstellar journey?

Chapter Three

Light as a Wave?

The whole of science is nothing more than a refinement of everyday thinking. It is for this reason that the critical thinking of the physicist cannot possibly be restricted to the examination of the concepts of his specific field. He cannot proceed without considering a much more difficult problem, the problem of analyzing the nature of everyday thinking.
— A. Einstein

SYNOPSIS

We examine attempts to measure the speed of light, note Newton's use of prisms to disperse light into various colours and review some of the properties of waves leading up to Young's double slit experiment and Maxwell's electromagnetic theory of light.

1. The Speed of Light

In the previous chapter we introduced the gravitational constant G as a universal constant fundamental to the gravitational force. It is our first example of a "*fundamental constant* in physics. We shall meet further fundamental constants in the next few chapters. Here we introduce a second fundamental constant, the speed of light c. The truly fundamental nature of the speed of light was not fully realised until Einstein's formulation of special relativity and his statement that observers in all inertial frames will report the *same* value for the speed of light in vacuum. Note that Einstein, like Galileo, goes to the simplest case, that of the vacuum. In doing this he eliminates all media effects such as refraction and dispersion. They are to be considered after the simplest case has been solved.

2. Measurement of the Speed of Light

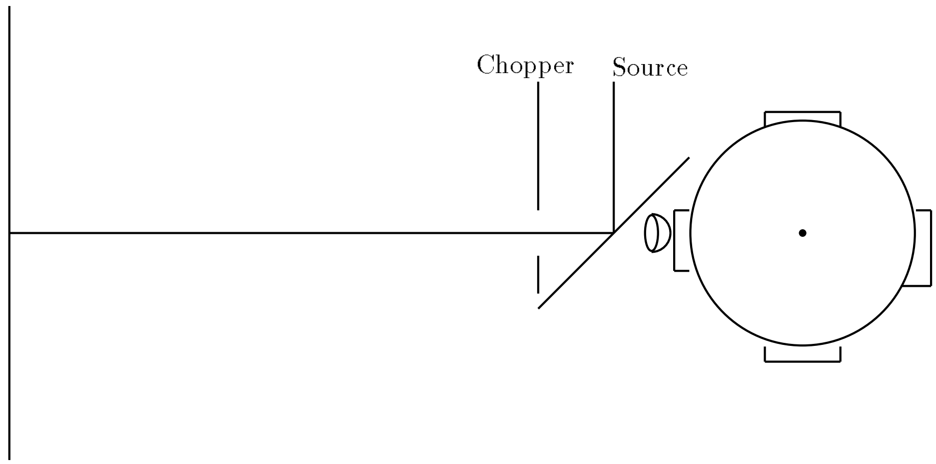
The first serious attempt to show that the speed of light is finite and to attempt to measure its speed was made by Galileo, unsuccessfully. Note, Galileo concluded from his failure that the speed of light was greater than he could measure - he did not conclude that the speed was infinite as had earlier persons. Galileo's method contained the basic idea of modern direct methods of measuring c, namely measuring the time taken for a light beam to travel from a source to a reflector and back to the source.

3. Roemer's Indirect Measurement of c

The first successful measurement of c was made by the Danish astronomer Ole Roemer who in the 1670's deduced a value from his study of the eclipses of the moons of Jupiter. He had noted that at a certain time of the year the moons reappear behind Jupiter about four minutes earlier than one would have expected from application of Newton's law directly. Six months later the moons appeared about four minutes late. Roemer realised that the difference in time arose from the fact that the Earth is closer to Jupiter when Earth is on the same side of the sun as Jupiter than when they are on opposite sides.

4. Direct Measurements of c

Subsequent measurements have all tended to involve the measurement of the time taken for a light beam to travel a measured distance. Basically a method is used to chop the beam into pulses which traverse a path of known distance. Thus in 1849 Fizeau used a rotating wheel cutting a beam of light on a 17km path.



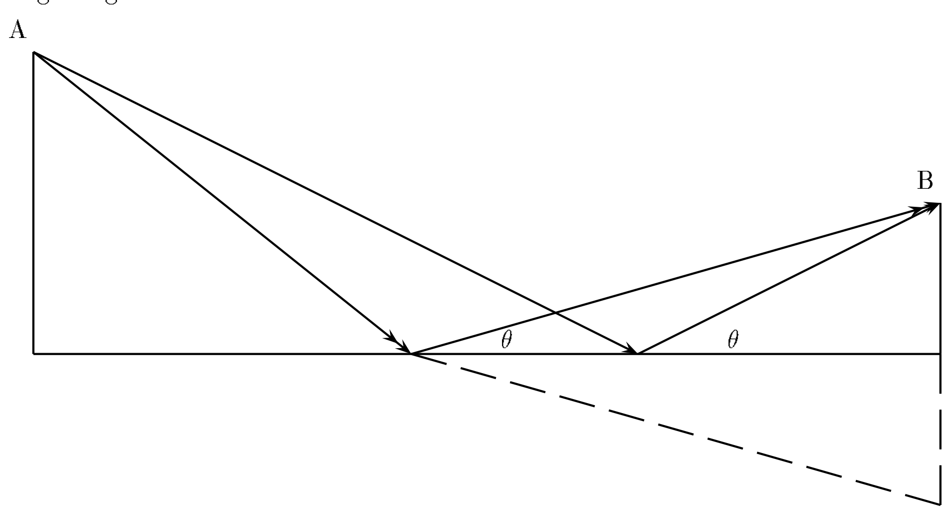
In 1850 Foucault completed a series of measurements that showed convincingly that the speed of light in water is *less* than in air. Great improvements in the measurement of c were made in the heroic experiments of the Polish/American physicist Michelson * (1852-1931). Present day measurements take all the advantages of modern technology, especially of electronics, resulting in the speed of light being one of the most precisely measured fundamental constants. Modern measurements give a value of

$$\boxed{c = 2.99792458 \times 10^8 \ ms^{-1}}$$

N.B. The gravitational constant is known to only 1 part in 10,000, the most imprecise of the so-called fundamental constants.

5. Reflection of Light

The law of reflection of light (angle of incidence = angle of reflection) was deduced by the Greeks long ago on the assumption that in travelling from a point A to a point B light takes the *shortest* distance and using the geometrical construction below arrived at the law of reflection.

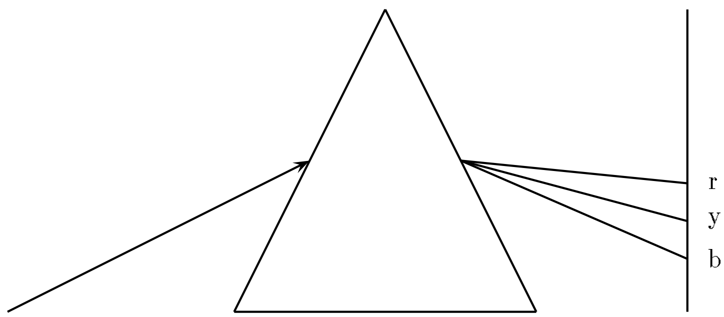


* Michelson was born in Strzelno, Poland in 1852. Visitors to Strzelno will find in the village square a plaque marking his birthplace and noting some of his achievements. The family migrated to the USA when Michelson was seven years old. Michelson's achievements were considerable - measurement of the speed of light, development of the Michelson interferometer, the famous null Michelson-Morley experiment that was crucial to the theory of relativity and the discovery of the fine structure of hydrogen that played and important role in relativistic quantum mechanics

Such a construction could not yield the corresponding laws of refraction. In the 1700's the *principle of least time* was introduced and led to both the laws of reflection and refraction could be derived by assuming that light takes the path of least time both in vacuum and in a medium.

6. Dispersion of Light

In the 1670's Newton commenced a series of experiments on the nature of light which were summarised in his remarkable book *Opticks*. Newton observed that white light could be broken up into a series of colours which always appeared in the same order. He observed that the *blue* light was bent most towards the base of the prism and the *red* light the least. This phenomenon is referred to as the *dispersion of light*.

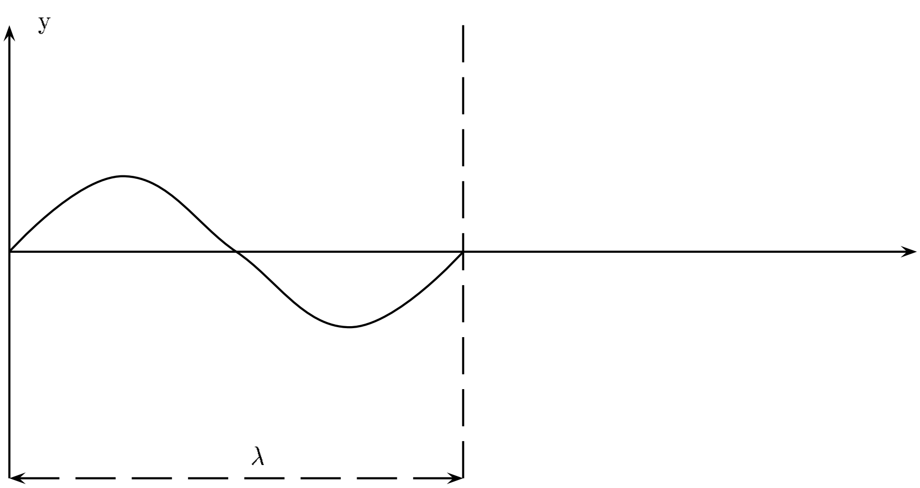


Newton assumed that light was made up of high speed particles (corpuscles). It appeared that the speed of light did not depend on the *intensity* of the light.

Christian Huyghens (1629-1695) put forward an alternative theory that a beam of light was a train of waves, a view largely ignored for a century.

7. Properties of a Wave

A wave represents a vibration, in the case of sound waves changes in pressures. A wave is said to be *periodic* if it regularly repeats its form over a period of time τ referred to as the *period* of the wave. The distance travelled in the time τ is known as the *wavelength* of the wave.



The *frequency* f of a wave is the number of oscillations that take place in a given time interval.

Since the time taken for one oscillation is equal to τ the period we have

$$\boxed{\boxed{f = \frac{1}{\tau}}}$$

Normally we will measure f in oscillations/second or equivalently cycles/second (cps). For a wave travelling at a speed c the distance covered in one period τ is a wavelength λ and hence

$$\lambda = c\tau$$

and

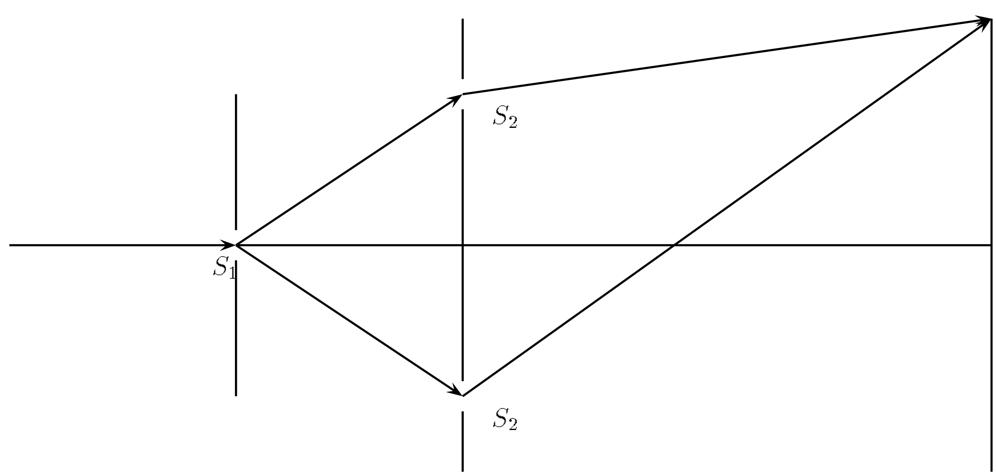
$$f = \frac{c}{\lambda}$$

8. The Wave Theory of Light

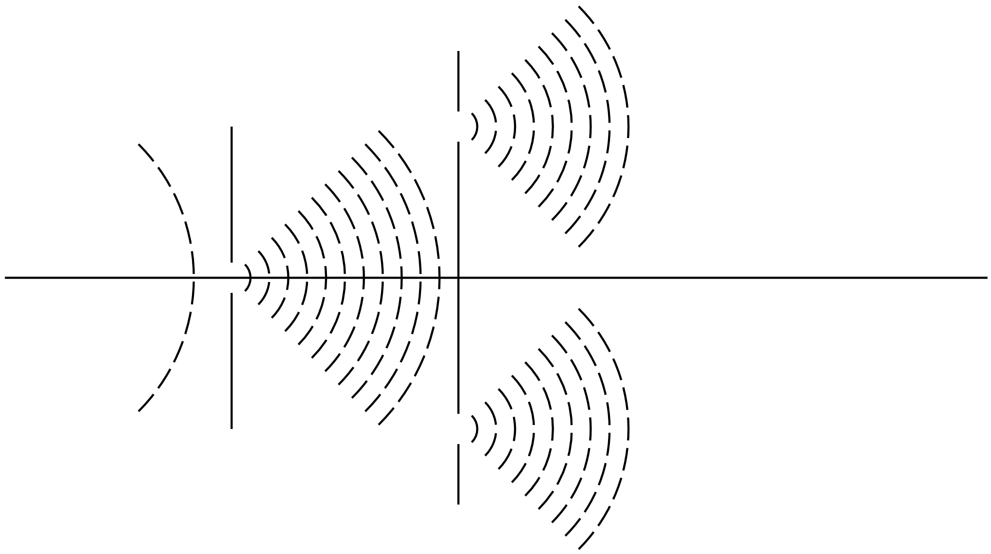
In 1801 Thomas Young (1773-1829) performed a number of experiments that seemed to only be compatible with a wave theory of light. Young suggested that two waves arriving at the same point may strengthen each other or cancel each other out. Strengthening would occur if the two waves were crest to crest whereas if the crest of one coincided with the trough of the other they would cancel out. Young was able to measure the wavelength of light and found that it depended on the colour of the light ranging from $0.4\mu m$ for violet to $0.8\mu m$ for red, i.e. 4×10^{-7} to 8×10^{-7} metres (c.f. size of an atom $\sim 10^{-10}$ metres).

9. Young's Double Slit Experiment

Young's wave description of light stemmed from his famous double slit experiment. The importance of Young's experiment cannot be overstressed. It was later to play a major role in the interpretation of quantum theory and remains today as an important device for understanding the foundations of quantum physics.



Huyghens had suggested that a wave striking a slit would spread out to form a series of advancing wave fronts as shown on the next page.



Young considered that if this wavefront impinged on two slits as shown then these two slits would act as the source of two new spherically expanding wavefronts. If the two wavefronts were intercepted by a screen then at some parts on the screen the wavefronts will reinforce one another leading to a bright line of light while at other parts the wavefronts would cancel leaving a dark line. The phenomena was termed *interference*. The interference of light was to lead to much modern technology as well as its scientific importance. Newton's corpuscular theory seemed incapable of describing such phenomena. As a result the wave theory of light was *the theory* for the next century.

Foucault's experiment showing that the speed of light was *less* in water than air was consistent with Young's wave theory of light but not with Newton's corpuscular theory.

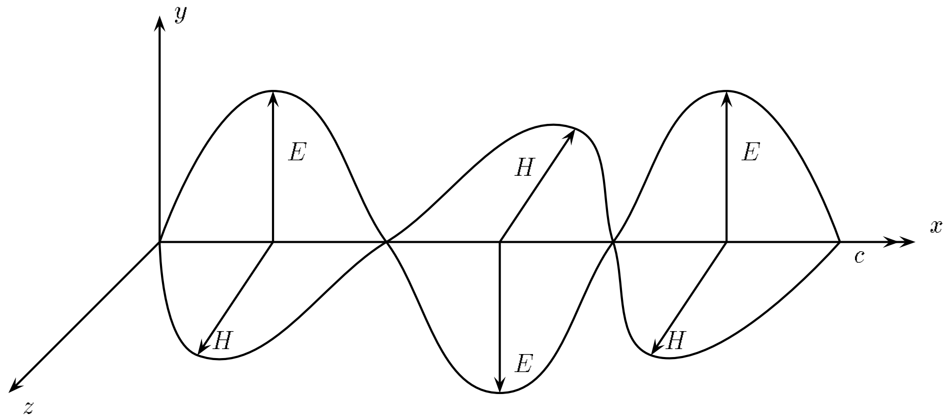
Note that in Young's experiment interference fringes are observed if both slits are open. Blocking one slit results in the disappearance of the interference pattern.

10. An Ether?

If light is a wave how does it propagate in a vacuum? Newton's answer was obvious but inconsistent with Young's interference experiments. Classically it was thought that the wave motion must involve a medium called the *ether*. All attempts to measure a change in the speed of light with respect to the ether failed - most notably in the Michelson-Morley experiment (1887).

11. Maxwell's Electromagnetic Theory of Light

It was known from the experimental works of Faraday (1791-1867) and of Oersted (1777-1851) that a changing magnetic field could create an electric field and vice versa. With that observation Maxwell (1831-1879) went on to predict the existence of *electromagnetic waves* - combinations of electric and magnetic fields that are continually oscillating and propagating through space free of matter, charge and current. Maxwell showed that the electric and magnetic fields were perpendicular to one another *and* perpendicular to the direction of propagation of the wave. A changing magnetic field creates an electric field perpendicular to the direction of the change of the magnetic field and vice versa.



12. The Speed of an Electromagnetic Wave

Maxwell showed that the electric and magnetic fields must change at such a rate that their speed is equal to a constant c. Remarkably, Maxwell was led to identify c with the speed of light. Maxwell concluded that light rather than being a mechanical vibration in an ether was an electromagnetic wave.

13. The Spectrum of Light

An electromagnetic wave is produced when electrons are accelerated. Maxwell's electromagnetic wave theory leads to all wavelengths of light being propagated at the *same* speed c in a vacuum. In a medium such as glass the speed of light c depends on the frequency f and hence it becomes possible to sort out different frequencies and hence wavelengths using a prism. Maxwell's theory places no restriction on the possible wavelengths or frequencies of light only on the speed. Thus we could expect to observe electromagnetic waves outside of the normal visible light range.

14. Why is the Study of Light Important?

Young realised that his discovery of the interference of light could lead to precise measurement of the wavelength of light. Michelson used the interference of light to develop what became known as the Michelson interferometer which played the key experimental evidence for the non-existence of the ether and perhaps more importantly was to lead to precise measurement of lengths in terms of wavelengths of light. The most precise measurements of lengths all involve the interference of light. The same interference phenomena are used in optical gyroscopes that allow pilots of 747 jumbo jets to determine their position with remarkable accuracy anywhere on their journey. Large optical gyroscopes are capable of measuring frequency shifts as small as one part in 10^{21} making possible studies of wobbles in the earth's rotation, measurement of rotational shears created by earthquakes etc. Interference of light is a key feature of laser technology and the use of light waves to transmit information at much higher densities than radio or microwave transmission. The understanding of the reflection and refraction of light has led to fibre optics which is revolutionising communications, surgery etc.

The ability to measure the wavelength of light was to lead to the whole subject of spectroscopy which in turn lead to the technologically productive quantum physics. Measurement of the Doppler effect associated with the relative velocity of sources and receivers of light was to enhance our knowledge of the universe by allowing us to determine the relative velocities of stellar objects as well as that of cars by using radar guns. Without the abstract study of the properties of light our world today would be a very different world and in my opinion a much poorer world.

Questions

Q1. You are running at the speed of light holding a mirror in front of you. What do you see in the mirror? (Einstein pondered this question when he was sixteen).

Q2. Why do the breast feathers of pigeons often appear to be changing mixtures of green and blue? (irridescent)

Q3. On a hot day in a desert telegraph poles often appear to be standing in water? (mirages)

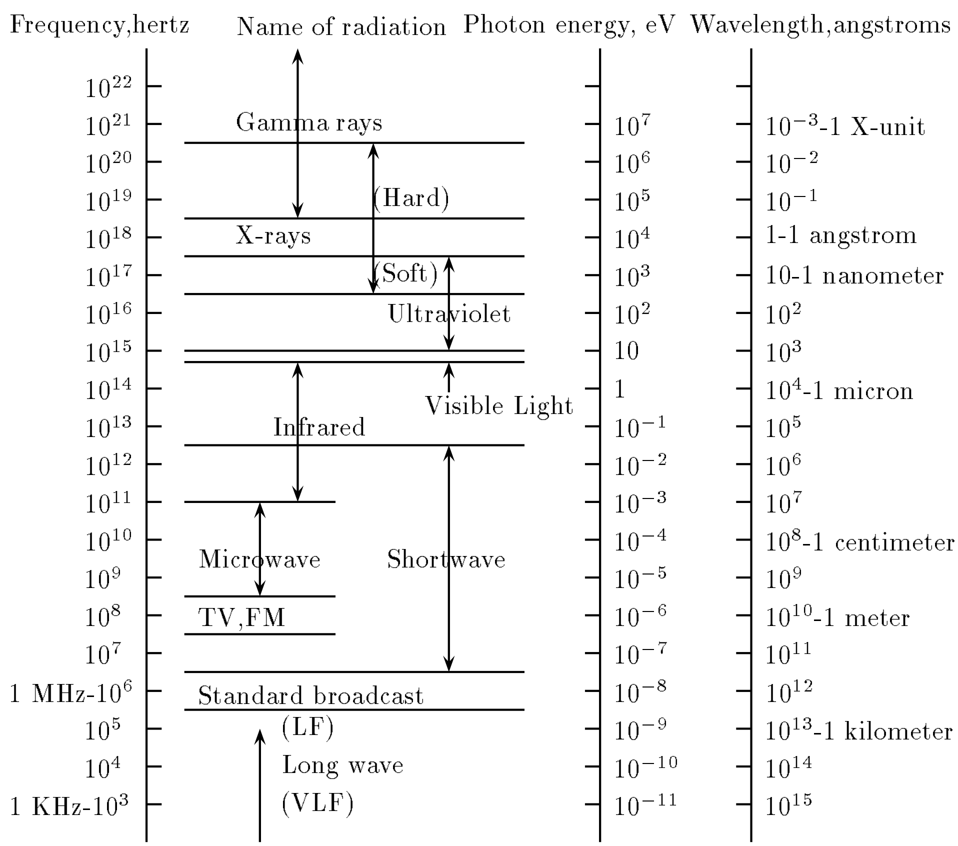
Q4. Why do people living in desert regions of the world where the daytime temperatures are very high often wear long loose fitting gowns?

Q5. Why is it possible to walk across a glowing hot bed of embers?

Two Experiments

1. Place your palm of your hand facing, within a few centimetres. your open mouth and exhale air upon it. What sensation do you feel on your hand?
2. Now repeat the experiment but this time with your lips pursed so that you must force the air out of your mouth. What sensation do you now feel on your hand?

Q6. Interpret the results of the above two experiments.



The whole of subject of electrical radiation seems working itself out splendidly.
— Oliver Lodge, Phil. Mag. (London) August 1888

Chapter Four

The Structure of Matter Begins

My new view of the first principles or elements of bodies and their combinations ... will produce the most important changes in the system of chemistry and reduce the whole to a science of great simplicity
— John Dalton (1766-1844)

SYNOPSIS In this chapter we first comment on Coulomb's law for the force between electric charges and then lead up to the discovery of the electron, X-rays and radioactivity.

1. Coulomb's Law of Electrostatics

The force between static electric charges was independently studied by Benjamin Franklin (1706-1790) and Charles Coulomb (1736-1806) leading to what is today known as Coulomb's Law of electrostatics. In modern notation and in SI units is written as

$$\boxed{F_C = k\frac{Q_1Q_2}{r^2}} \tag{1}$$

where the charges Q_1, Q_2 are measured in Coulombs, their separation in metres and

$$\boxed{k = \frac{1}{4\pi\epsilon_0} \sim 9 \times 10^9} \tag{2}$$

with

$$\epsilon_0 = 8.854187817 \times 10^{-12} Fm^{-1}$$

being the permittivity of free space. Note the very close similarity between Coulomb's law and Newton's gravitaional force equation. An indication of the relative strengths of gravitational and electrical forces can be found by calculating the ratio of the forces for two electrons placed a distance r apart. Then

$$\begin{aligned}\frac{F_C}{F_G} &= \frac{k}{G}\left(\frac{e}{m}\right)^2 \\ &= \frac{9 \times 10^9}{6.7 \times 10^{-11}(1.76 \times 10^{11})^2} \\ &=\sim 4.3 \times 10^{43}\end{aligned} \tag{3}$$

which is a dimensionless number that does not depend on the system of units used. Equation (3) demonstrates that electric forces are vastly stronger than gravitational forces. Both are inverse-square forces of apparently infinite range. Significantly the gravitational force is a purely attractive force whereas the electric force can be attractive (charges of opposite sign) or repulsive (charges of the same sign). The gravitational force is additive, the more mass the stronger the attraction whereas electrical forces largely cancel out due to the charge on protons being, to astonishing precision, of the same magnitude but opposite sign to that of the electron . Intuitively we tend to conclude from our daily experience that the gravitational force dominates and yet it is the weakest of all known forces. This is because most systems we encounter are electrically neutral - there are an equal number of positive and negative charges. If one were to place all the negative charges of an average person at a point separated from their positive charges placed at another point so that they are separated by one metre we would find an attractive force between them of the order of the weight of the earth.

2. Chemical Elements

The Greeks recognised four elements - earth, water, air and fire. Boyle(1627-1691) and Lavoiser (1743-1794) developed the modern concept of a *chemical element* as a substance that could not be divided into two or more different substances. This spelt the death knell of alchemy. Gold was identified as an element. Chemical compounds were resolved into specific combinations of chemical elements.

John Dalton*(1765-1844) postulated that each element consisted of atoms which were all alike, immutable. Each element involved different atoms. This effectively established chemistry as the science involving combinations of atoms.

3. Atomic Masses

It became possible to prepare samples of different elements that possessed the same number of atoms. Comparison of different samples having the same number of atoms led to the concept of *atomic mass*. If the atomic mass of hydrogen was taken as 1 then the atomic mass of helium was found to be 4, carbon 12, oxygen 16, etc.

4. Molecules and Molecular Masses

Molecules were assumed to be composed of definite combinations of atoms, e.g. the water molecule H_2O. The *molecular mass* of a given molecule was essentially the sum of the atomic masses of its constituents.

$$\text{e.g. } H_2O \text{ molecular mass } 2 + 16 = 18$$

Note: Mass *numbers* are integers but *molecular masses* are only approximate integers.

5. Atomic and Molecular Weights

The gram atomic or gram atomic weights are just the atomic or molecular masses expressed in grams. Thus the gram atomic weight of of oxygen is 16gms while the gram molecular weight of water H_2O is 18gms.

6. Avogadro's Number

The number of atoms in a gram atomic weight or molecules in a gram molecular weight is a fixed number known as *Avogadro's number* N_A named after Avogadro (1776-1856) which was first determined by Loschmidt (1821-1895). Its modern value is

$$\boxed{N_A = 6.00221367(86) \times 10^{23}\ mol^{-1}}$$

N.B. This is an enormous number!

7. The Classical Atom

The atom as conceived by Dalton and his followers lacked any structure and was assumed to be an indestructable, indivisible particle. No theoretical basis existed to interpret their atomic weights or their modes of combination. Their structure was to be revealed in the twentieth century.

* The school master John Dalton was colour blind to the extent of only being able to distinguish shades of grey. That is the origin of the term *Dalton blindness*

8. The Cathode Ray Tube

Electrical discharges were observed in evacuated glass tubes with a high voltage applied between the cathode and anode as above.

1833 **Faraday** (England) studies electrical discharges in gases "*rarefaction of the air wonderfully favours the glow phenomena*".

1858 **Plúcker** (Germany) observes that the glow is affected by a magnetic field.

1868 **Hittorf** (Germany) uses a mercury vacuum pump to obtain a higher vacuum and sees a shadow cast by an object placed in front of the cathose indicating that the discharge originated in the cathode.

1879 **Crookes** (England) achieves a higher vacuum with improved pumps. Rays appear to emanate from the cathode and travel down the tube - CATHODE rays.

1892 **Hertz** (Germany) claims experimental evidence that the cathode rays are waves - all German physicists agree.

1892 **Crookes** (England) claims cathode rays are radiant electrically charged matter - all English physicists agree.

1894 **Stoney** (England) coins the name *electron*.

1895 **Perrin** (France) proves that the rays are *negatively* charged particles - discovery of the electron.

1897 **J. J. Thompson** (England) confirms the corpuscular nature of cathode rays and measures their velocity and the ratio of their charge e to their mass m.

1899 **Thompson** and **C. T. R. Wilson** measure separately the charge e and the mass m of the electron.

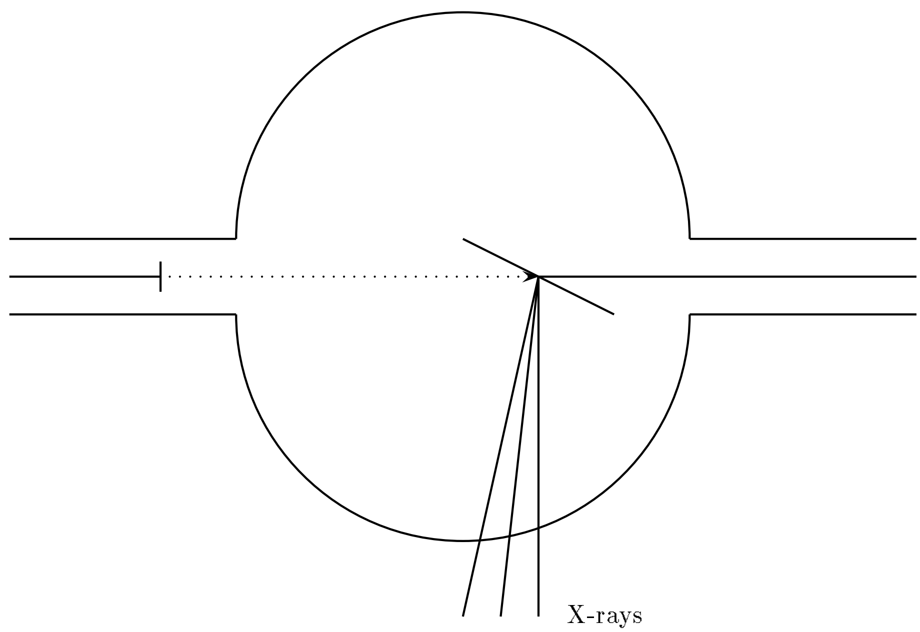
$$\boxed{e = -1.602189 \times 10^{-19} \text{ Coulombs}}$$

$$\boxed{m = 9.10953 \times 10^{-31} \text{ kilograms}}$$

The discovery of the electron represented the first isolation of a fundamental particle and was to usher in the electronic age. The fact that the cathode rays could be manipulated by electric and magnetic fields was to very shortly lead to the first primitive attempts at producing television. Note that television did not arise as a result of market demand for a new product but was the final end product of a long series of basic discoveries in science. Each step, apparently unrelated to the final product, was essential and totally unplanned. This we need to bear in mind in thinking about how new technology arises and how its development is to be stimulated.

9. Röntgen's X-rays

The discovery of the electron was largely overshadowed by Röntgen's discovery of X-rays in 1895. Röntgen's discovery was to have a vast impact on the nature of atoms and lead ultimately to Becquerel's amazing discovery of radioactivity. Röntgen's discovery was totally unexpected and came from Röntgen's curiosity concerning the observation that fluorescence was often seen on the glass envelope of a cathode ray tube. Röntgen had used a large induction coil to create the discharge in the tube. Paper covered with barium platino-cyanide was used in the detection of fluorescence. He noted that if the apparatus was placed in a dark room and covered fluorescence of the paper could still be detected each time the induction coil discharged. He was struck by the observation that fluorescence occurred even when the coated side of the paper was not facing the tube. He realised some radiation must be penetrating the paper. He then observed the effect persisted even if a double pack of cards was placed in front of the paper. Finally, to his amazement, he saw the shadow of the bones of his hand on a screen.



Almost immediately the medical possibilities of X-rays were realized. The guess work of bone setting was eliminated.

10. H. Becquerel's Discovery of Radioactivity (1896)

Röntgen's discovery created a sensation. On 20 January 1896 Henri Poincaré shows Henri Becquerel one of Rόntgen's X-ray photographs. Becquerel thinks that since the glass where the X-rays emerge fluoresces that a relationship must exist between fluorescence and X-rays.

30 Jan 1896 Poincaré asks *"Do all bodies whose fluorescence is sufficiently intense emit both luminous rays and also Rόntgen's X-rays, whatever the cause of their fluorescence?"*

24 Feb 1896 Henri Becquerel reports to the Academie des Sciences, Paris on his experiment involving a uranium salt, uranyl potassium sulphate, known to fluoresce. *I wrapped a photographic plate with two sheets of thick black paper, so thick that the plate did not become fogged by exposure to the sun for a whole day. I placed on the paper a layer of the phosphorescent substance, and exposed the whole thing to the sun for several hours. When I developed the photographic plate I saw the silhouette of the phosphorescent substance in black on the negative... The same experiment can be tried with a thin sheet of glass placed between the phosphorescent substance and the paper, which excludes the possibility of a chemical action resulting from vapours that might emanate from the substance when heated by the sun's rays. We may therefore conclude from these experiments that the phosphorescent substance in question emits radiations that penetrate paper that is opaque to light...*

The 26/27th February the weather was poor and the sun did not appear for long enough to repeat the experiment. Becquerel leaves the prepared plates in a drawer.

2 Mar 1896 Becquerel reports again to the Academie. *Since the sun did not show itself again for several days, I developed the photographic plates on the 1st of March, expecting to find images very feeble. On the contrary, the silhouettes appeared with great intensity. I thought at once that the action might be able to go on in the dark.*

9 Mar 1896 Becquerel found that the radiation emitted by the uranium not only blackened photographic plates but also ionized gases making them conductors.

Studies showed that the radiation emitted was independent of the chemical form of the uranium showing that it was directly associated with the uranium.

11. Radium and Polonium

Marie Skłodowska-Curie*** (1867-1934) and Pierre Curie (1859-1906) succeeded (1898) in isolating two new radioactive elements, radium and polonium, ${}^{226}_{88}Ra$ was found to have a *half-life* of $\sim 1600yr$ and ${}^{210}_{84}Po$ of $\sim 138.4d$ and which thus vastly more radioactive than ${}^{238}_{92}U$ ($t_{\frac{1}{2}} = 4.51 \times 10^9 y$) or ${}^{235}_{92}U$ ($t_{\frac{1}{2}} = 7.13 \times 10^8 y$).

12. Matters of Notation

Half-Life The half-life of a radioactive atom is the time taken for one half of a sample to have decayed by a given process. **Note** That does NOT mean half the sample has disappeared, rather half the atoms have decayed.

Becquerel The standard measure of radioactivity is the Becquerel (Bq) and corresponds to one decay/sec.

Activity The *activity* $\mathcal{A}$ of a radioactive sample was found experimentally to be proportional to the number N of radioactive atoms in the sample. Thus

$$\boxed{\boxed{\mathcal{A} = \lambda N}} \tag{1}$$

The proportionality constant λ is known as the *decay constant.*

Exponential Decay Let N_0 be the number of radioactive atoms of a given isotope at $t = 0$. The number of disintegrations/second is

$$\boxed{\boxed{-\frac{dN}{dt} = \lambda N}} \tag{2}$$

Integration of Eq.(2) leads to the exponential law of radioactive decay

$$\boxed{\boxed{N = N_0 e^{-\lambda t}}} \tag{3}$$

Putting $N = \frac{N_0}{2}$ in Eq.(3) leads to

$$\boxed{\boxed{t_{\frac{1}{2}} = \frac{1}{\lambda} ln2 = \frac{0.693}{\lambda}}} \tag{4}$$

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* Marie Skłodowska-Curie is usually viewed as the heroine who with tremendous energy and labour separated out the element radium from pitchblende - true but often we overlook what was probably her most important discovery the explicit recognition that radioactivity is associated with individual atoms and that such atoms are inherently unstable and decay statistically rather than deterministically.

** Marie Skłodowska-Curie was the first woman to given burial in the French Panthéon. It has commonly been thought she died of the effects of exposure to radium, however when her body was exhumed for reburial the level of radium in her coffin was found to be well below maximum accepted levels for public exposure. It is now thought that her death was associated with her considerable exposure to X-rays during her work with injured soldiers during World War I.

13. Types of Radioactive Decay

Rutherford (1871-1937) in 1897 found that the radiation from radioactive materials was more complex than just X-rays. Three distinct types of radioactive decay were identified:

$\alpha-$decay $\alpha-$particles are emitted and correspond to the **nuclei** of helium atoms (i.e. a tightly bound cluster of two protons and two neutrons). The particles carry two units of positive charge. They are strongly ionising particles - stopped by a sheet of paper. Very short penetration - dangerous if ingested.

$\beta-$decay Energetic electrons (or positrons) - less ionising than $\alpha-$particles - penetrate tissue orders of millimetres.

$\gamma-$decay Electromagnetic radiation similar to visible light but of much shorter wavelength. Similar to X-rays but of nuclear origin. More penetrating than $\beta-$particles.

14. The Electron Volt

In succeeding lectures we will use the *electron volt* eV as an energy unit. One electron volt is the amount of energy acquired by an electron moving through a potential difference of one volt.

$$\boxed{\boxed{1eV = 1.6021 \times 10^{-19} Joules}}$$

Multiples of the electron volt find much use in nuclear and particle physics. Thus

$$\begin{aligned}
1kiloelectronvolt &= 1keV = 10^3 eV \\
1Megaelectronvolt &= 1MeV = 10^6 eV \\
1Gigaelectronvolt &= 1GeV = 10^9 eV \\
1Teraelectronvolt &= 1TeV = 10^{12} eV
\end{aligned}$$

Chemical bonding energies are $\sim 10eV$, X-rays occur in the keV range whereas typical nuclear transitions are $\sim 1 - 10MeV$. The largest particle accelerators involve particle energies $\sim TeV$.

Questions

Q1. How big is 10^{43}? You have a supercomputer that can count up to 10^9 in one second. How long would it take to count to 10^{43}?

Q2. What would be the cosmological consequences of a difference in the magnitude of the charges on the proton and electron being slightly different?

Q3. Explain why Mass *numbers* are integers but *molecular masses* are only approximate integers.

Q4. Estimate the number of molecules in a 60kg person.

Q5. The human body contains radioactive K^{40}_{19} giving rise in the average human to about 4000decays/sec. Each decay involves an energy of about $10^6 eV$. If it takes approximately $10eV$ to break a chemical bonds what is the maximum number of chemical bonds that could be broken during a one hour lecture? Compare you answer with the number obtained in the previous question.

Q6. The year is 1830. The Medical Congress passes a resolution seeking the setting up of a fund to improve the treatment of bone fractures. Would you consider making grants to assist the researches of Faraday, Hertz, Crookes, Perrin, or Röntgen?

Q7 The human body contains 0.2% of potassium. Estimate the number of atoms of K in a $70kg$ person.

Q8 Naturally occuring potassium contains 0.0118% radioactive ${}^{40}_{19}K$. How many of the atoms calculated in Q7 are radioactive?

Q9 ${}^{40}_{19}K$ has a half-life of $1.28 \times 10^9 y$. What is the activity $\mathcal{A}$ for a $70kg$ person in Bq?

...Not that I aspire to complete coherence. Our mistake is to confuse our limitations with the bounds of possibility and clap the universe into a rationalist hat or some other. But I may find the indications of a pattern that will include me, even if the outer edges tail off into ignorance.

— William Golding, *Free Fall (1959)*

Chapter Five

The Stability of Matter

The universe is infinite in all directions, not only above us in the large but also below us in the small. If we start from our human scale of existence and explore the content of the universe further and further, we finally arrive, both in the large and in the small, at misty distances where first our senses and then even our concepts fail us
— Emil Wiechert, Königsberg (1896)

1. SYNOPSIS

The discovery of radioactivity revealed that certain elements are unstable. The concept of the absolute stability of matter was lost. In this chapter we develop the concept of the stability of matter and its limitations. After briefly considering some matters of notation we commence by first looking at natural radioactivity and then nuclear reactions and the production of new elements, nuclear fission and fusion. The Oklo phenomena is then discussed and finally the question "Are the ultimate constituents of matter stable?"

2. Review of Notation

Atomic Number Z

The different elements may be distinguished by their **atomic number** Z which is equivalent to the number of protons contained in the nucleus and gives the number of units of positive charge in the nucleus. Neutral atoms have a core (or nucleus) of positive charge Ze surrounded by Z electrons.

Neutron Number N

The atoms of a given element, X, contains an integer number, N, of neutrons each of approximately the same mass as the proton but carrying no electric charge.

$$\boxed{m_p = 1.6725 \times 10^{-27} kg \qquad m_n = 1.6748 \times 10^{-27} kg}$$

or

$$\boxed{m_p = 938.272 MeV/c^2 \qquad m_n = 939.565 MeV/c^2}$$

Nucleon Number A

Neutrons and protons are collectively referred to as **nucleons**. The nucleon number A is equal to the sum of the neutron (N) and proton (Z) numbers.

$$\boxed{A = Z + N}$$

Isotopes

Several values of N may be associated with a given element, X, of atomic number Z and are referred to as **isotopes** of the element X. The isotopes of an element X will normally be designated as

$$\boxed{{}^{A}_{Z}X}$$

e.g.

$${}^{4}_{2}He \qquad {}^{3}_{2}He$$

N.B. The α−particle is a helium nucleus, ${}^{4}_{2}He$.

$\alpha-$decay

$$ {}^{A}_{Z}X \xrightarrow{\alpha} {}^{A-4}_{Z-2}Y + {}^{4}_{2}He $$

$$ \text{e.g.} \quad {}^{238}_{92}U \xrightarrow{\alpha} {}^{234}_{90}Th + {}^{4}_{2}He $$

N.B. The nucleon number A is *decreased* by four units and the atomic number Z decreased by two units.

3. $\beta-$decay

Three distinct types of $\beta-$decay are observed.

$$ {}^{A}_{Z}X \xrightarrow{\beta^-} {}^{A}_{Z+1}Y + e^- + \bar{\nu} \quad \textit{electron emission} $$

$$ \text{e.g.} \quad {}^{40}_{19}K \xrightarrow{\beta^-} {}^{40}_{20}Ca + e^- + \bar{\nu} $$

$$ {}^{A}_{Z}X \xrightarrow{\beta^+} {}^{A}_{Z-1}Y + e^+ + \nu \quad \textit{positron emission} $$

$$ \text{e.g.} \quad {}^{40}_{19}K \xrightarrow{\beta^+} {}^{40}_{18}Ar + e^+ + \nu $$

$$ {}^{A}_{Z}X + e^- \xrightarrow{EC} {}^{A}_{Z-1}Y + \nu + \gamma \quad \textit{electron capture} $$

$$ \text{e.g.} \quad {}^{40}_{19}K + e^- \xrightarrow{EC} {}^{40}_{18}Ar + \nu + \gamma $$

Note that a given isotope may decay by more than one process or mode, as shown, for example by ${}^{40}_{19}K$ above. The nucleon number N is unchanged but the atomic number Z is *increased* or *decreased* by one unit. The positron is the antiparticle of the electron.

4. Natural Radioactive Decay

A number of radioactive isotopes occur in nature. Thus uranium occurs in the earth's crust at about 4μgm in each gram of rock and in sea water betweem 0.3 and $2.3\mu gm$ per litre. Thus a sea water Olympic swimming pool $100 \times 20 \times 2$metres would contain one or two grams of uranium.

isotope	half-life	abundance	
${}^{238}_{92}U$	$5.4 \times 10^9 y$	99.28%	
${}^{235}_{92}U$	$7.1 \times 10^8 y$	0.715%	(1)

5. Naturally Occurring ${}^{40}_{19}K$

0.0118% of all potassium occurs as the radioactive isotope ${}^{40}_{19}K$ with a half-life of $1.3 \times 10^9 y$. The occurrence of radioactive potassium along with thorium in the earth supplies a very significant heat output.

6. Nuclear Reactions

The first man-made nuclear reaction was produced by Rutherford in 1919 when he bombarded nitrogen with $\alpha-$particles.

$$ \boxed{{}^{14}_{7}N + {}^{4}_{2}He \rightarrow {}^{17}_{8}O + {}^{1}_{1}H} $$

The first nuclear reaction producing a radioactive nuclide was made in 1934 by Irene Joliot Curie and her husband Frédéric

$$ {}^{27}_{13}Al + {}^{4}_{2}He \rightarrow {}^{30}_{15}P + {}^{1}_{0}n $$

$$ {}^{30}_{15}P \xrightarrow{2.5min} {}^{30}_{14}Si + e^+ + \nu $$

7. Discovery of the neutron

In 1932 J. Chadwick bombarded ${}^{9}_{4}Be$ with $\alpha-$particles and observed the reaction

$$ \boxed{{}^{9}_{4}Be + {}^{4}_{2}He \rightarrow {}^{12}_{6}C + {}^{1}_{0}n} $$

Chadwick inferred the existence of the neutron and deduced its mass must be slightly greater than that of the proton. The neutron itself was observed to be unstable in free space with a half-life of $sim10min$.

$$\boxed{{}_0^1n \rightarrow {}_1^1 p + e^- + \bar{\nu}}$$

Nevertheless bound in a nucleus the lifetimes of the proton and neutron are the same.

8. New Elements and the Discovery of Nuclear Fission

In 1934 Fermi, Amaldi, D'Agostino and Rasetti started to irradiate atoms with neutrons to create new isotopes. Within 3 years they had produced 40 new radioactive isotopes using a $Ra - Be$ neutron source.

$${}_{88}^{226}Ra \xrightarrow{\alpha} {}_{86}^{222}Rn + {}_2^4 He$$

$${}_4^9Be + {}_2^4 He \rightarrow {}_6^{12} C + {}_0^1 n$$

Fermi suggested neutrons could be used to produce new elements and bombarded ${}_{92}^{238}U$ with neutrons

$${}_{92}^{238}U + {}_0^1 n \rightarrow {}_{92}^{239} U + \gamma$$

$${}_{92}^{239}U \xrightarrow{\beta^-} {}_{93}^{239}Y + e^- + \bar{\nu}$$

Fermi and collaborators sought to produce a new element Y and succeeded in producing new unidentified radioactive substances. In 1935 Ida Noddack criticised Fermi's experiment pointing out that Fermi had not proved that uranium could not break up into two large fragments.

In late 1934 Fermi observed that nuclear reactions involving neutrons seemed to occur more efficiently if the neutrons were passed through a block of paraffin and correctly concluded that the neutrons were slowed down by elastic collisions. Within a few hours he and his collaborators concluded the experiment and on the evening of 22 October 1934 wrote a one page paper.

In 1939 Hahn and Strassman found radioactive barium among the products of bombardment of uranium with slow neutrons and reported

> *As a consequence of these investigations we must change the names of the substances mentioned in our previous disintegration schemes, and call what we previously called radium, actinium, and thorium, by the names barium, lanthanum, and cerium. As nuclear chemists who are close to the physicists, we are reluctant to take this step that contradicts all previous experiences of nuclear physics.* Naturwissenschaften **27**11 (1939).

This was the discovery of a new type of nuclear decay - *nuclear fission*. It was found that ${}_{92}^{238}U$ could be made to fission only with fast neutrons whereas ${}_{92}^{235}U$ could fission with slow neutrons.

9. Production of Transuranic Elements

In 1940 McMillan and Abelson identified the element Fermi had been trying to create from bombarding uranium with neutrons.

$${}_{92}^{238}U + {}_0^1 n \rightarrow {}_{92}^{239} U \xrightarrow{23min} {}_{93}^{239}Np + e^- + \bar{\nu}$$

and soon thereafter the element plutonium via

$${}_{93}^{239}Np \xrightarrow{2.3d} {}_{94}^{239}Pu + e^- + \bar{\nu}$$

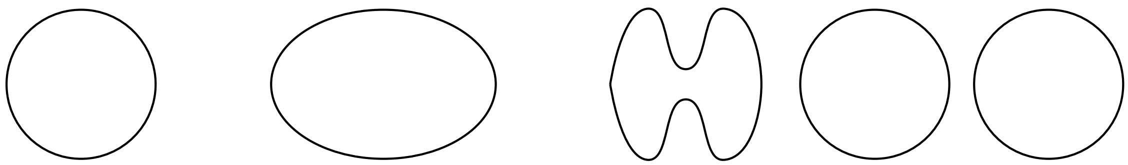
which was also found to be fissile and with a half-life of $24400y$. These discoveries led ultimately to the discovery of all the elements of the transuranic series.

10. Nuclear Fission

Nuclear fission involves the deformation of the nuclei of heavy elements leading to the separation of the nuclei into two fragments. The absorption of an extra neutron by ${}_{92}^{235}U$ results in

$${}_{92}^{235}U + {}_0^1 n \rightarrow {}_{92}^{236} U^*$$

which is an unstable isotope of uranium that is sensitive to deformation of its nuclear shape



spherical nucleus   deformed nucleus   fissioning nucleus   fission fragments

The nett effect is to produce two very energetic fragments together with *extra neutrons* and $\gamma-$rays. The fission fragments are highly radioactive and will gradually decay into stable elements.

11. Nuclear Chain Reactions

The extra neutrons created in the fission process can be used to induce more fissions and hence more neutrons and more fissions. If the sample is too small the mass will be *subcritical* and the chain reaction will not be self-sustaining. If the mass is sufficiently large the chain reaction can be self-sustaining and the mass is said to be *critical*.

12. Nuclear Fusion

In nuclear fission energy is released when nuclei split to form lighter nuclei. It is also possible to build-up light atoms from lighter atoms and produce energy. Thus if two protons react we can have

$${}^1_1H + {}^1_1H \rightarrow {}^2_1H + e^+ + \nu$$

leading to the formation of deuterium ${}^2_1H$. Such a reaction is strongly hindered by the Coulomb repulsion of the two protons approach one another prior to fusion.

The reactions

$$\begin{aligned} {}^2_1H + {}^2_1H &\rightarrow {}^3_2He + {}^1_0n \\ {}^2_1H + {}^2_1H &\rightarrow {}^3_1H + {}^1_1p \\ {}^3_1H + {}^2_1H &\rightarrow {}^4_2He + {}^1_on \end{aligned}$$

all give rise to considerable excess energy known as *fusion* energy. These reactions are basic to *thermonuclear reactions*.

13. Solar Energy

Fusion energy rather than fission energy is the source of solar energy. This is due to the abundance of light elements in the sun. The proton-proton chain

$$\begin{aligned} {}^1_1H + {}^1_1H &\rightarrow {}^2_1H + e^+ + \nu \\ {}^2_1H + {}^1_1H &\rightarrow {}^3_2He \\ {}^3_1He + {}^3_2He &\rightarrow {}^4_2He + 2\,{}^1_1H \end{aligned}$$

is largely responsible for energy production in the sun and similar small stars. In larger mass stars a carbon-nitrogen cycle is involved.

14. The Oklo Phenomena

A knowledge of the half-life of an element allows us to predict the future of a radioactive sample. Thus the half-life of ${}^{235}_{92}U$ is $7.1 \times 10^8 y$ and hence a $1kg$ sample of ${}^{235}_{92}U$ will after 7.1×10^8 years contain only $500gm$ of ${}^{235}_{92}U$ the rest being various decay products. If we observe today a $1kg$ sample of ${}^{235}_{92}U$ in an ore then presumably 7.1×10^8 years ago it contained $2kg$ of ${}^{235}_{92}U$. Working backwards in time we have for the abundance of ${}^{235}_{92}U$ as a percentage of the total uranium content

Years before present time	$\%{}^{235}_{92}U$
0	0.72%
0.5×10^9	1.08%
1×10^9	1.63%
1.5×10^9	2.44%
2×10^9	3.65%

Thus the concentration of ${}^{235}_{92}U$ in the earth was higher than at the present time. P. K. Kuroda published a paper *J. Chem. Phys.* **25, 781 (1956)** entitled *Could nuclear reactors have spontaneously started up on earth* $\sim 2 \times 10^9$ *yrs ago?*. In 1972 in Oklo, Gabon irrefutable evidence was found for the occurrence there of a natural nuclear reactor. For a reviews see R. Naudet, Interdisciplinary Science Reviews, **1**, 72 (1976) and the book *The Oklo Phenomena*, I.A.E.A. Vienna (1975).

15. Is Matter Forever?

What do we mean when we say an atom or particle is stable? We can never answer the question with certainty but we can establish experimental limits. When we say something is stable we are saying no experiment, as yet, has observed any decays. Many isotopes have half-lives that are many powers of ten longer than the age of the universe. There are good theoretical reasons to suppose that electrons and neutrinos are absolutely stable. The ultimate stability of matter will depend on whether protons decay into lighter particles such as positrons, mesons etc. Some unified theories have predicted that protons should decay with a half-life of $\sim 10^{31}$ years that is about 10^{21} times longer than the age of the universe. Current experimental results give $t_{\frac{1}{2}} > 10^{32}$ years.

How can we measure such an enormously long lifetime?

1gm of matter contains $\sim 6 \times 10^{23}$ nucleons

1 tonne $= 10^6 gm$

1 tonne contains 6×10^{29} nucleons

1000 tonnes contains 6×10^{32} nucleons

Thus if $t_{\frac{1}{2}} = 10^{32}$ years then we expect a 1000tonne sample to yield approximately 6 decays/yr. For more details see S. Weinberg, *Proton Decay*, Scientific American June (1981).

16. Questions

Q1 Is antimatter ever produced in the human body?

Q2 If the present percentage of ${}^{235}_{92}$ is 0.715% what would the percentage have been 2×10^9 years ago?

Q3 Could a nuclear reactor have spontaneously started up on earth $\sim 2 \times 10^9$ years ago?

Q4 The man-made element ${}^{243}_{95}Am$ decays by α emission and is commonly used in smoke detectors in houses. Explain how such a device works.

Q5 Can the Oklo phenomena tell us how nuclear waste can move over geological periods of time?

Ceci's ego was not destroyed or transformed by her relationship with God but compressed, squeezed, compacted, to the very point of collapsing under its own mass. The gravitational force of her passion had become so strong in her that light could not escape outwards, and she gave little joy or comfort to those around her. She had become a black hole: there was a perfect dark night of her soul. Time slowed down in that gravitational force; as if in slow motion once the irrevocable process had begun she was stretched, twisted, warped by the plunge; and everything that was Ceci was sucked towards the singularity; and thence into an alternative universe beyond the laws of physics and psyche, into a new universe, into God.

– Sara Maitland *Home Truths,*Chatto and Windus, London (1993)

Chapter Six

Black-Body Radiation

Lectures were once useful; but now, when all can read, and books are so numerous, lectures are unnecessary. If your attention fails, and you miss a part of a lecture, it is lost; you cannot go back as you do upon a book.
— Samuel Johnson, 15th April 1781 : see Boswell *Life of Johnson, London (1791)*

1. SYNOPSIS

We explore the subject of black-body radiation which led to the development of quantum theory and provided a means of determining the temperature of stars and furnaces. The black-body radiation is of fundamental significance in Big Bang cosmology.

2. Absorption and Emission of Radiation

The rate at which a body emits or absorbs radiant energy depends on its absolute temperature and the nature of its exposed surfaces. Objects that are good emitters are also good absorbers of the same kind of radiant energy. A blackened body is an excellent emitter as well as an excellent absorber.

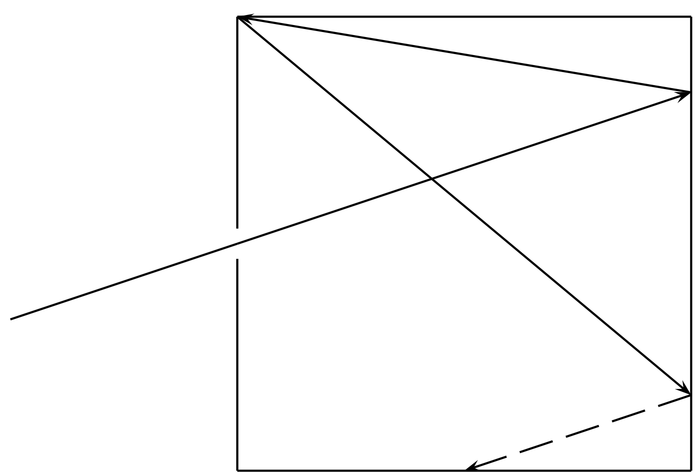
3. Change of Colour with Temperature

It has been known, almost from earliest times, that a metallic rod will, with increasing temperature, start to glow red and as it gets still hotter become yellow and thence white. This shift of colour seems to be largely independent of the nature of the rod.

4. The Ideal Black-Body

In 1859 Kirchoff introduced the idea of a perfect black-body as a body that completely absorbs all wavelengths λ of radiation that falls on it. Such a body must also be a perfect emitter of radiation.

5. The Black-Body Cavity



Consider a box, as above, having black walls. A light beam entering the box will be partially reflected and partially absorbed each time it strikes the walls. After a number of reflections the light will be totally absorbed even if the walls were not perfectly black. To a person looking from the outside, at room temperature, the aperture in the box will appear blacker than black!

6. Radiation from a Black-Body

Imagine a black-body cavity is heated so that the walls are kept at a constant temperature of $T^o K$. The walls will emit radiation. Since the cavity behaves as a perfect absorber it must also be a perfect emitter independently of the precise nature of the walls. The whole cavity will be filled with radiant energy. If the cavity contains a small aperture radiant energy will stream from the aperture. The aperture behaves as an ideal black-body emitting radiation exactly as for an ideal black-body of the same temperature as the cavity and its walls.

We can measure the total energy E emitted in one second by the aperture in Joules and if we divide this energy by the area A of the aperture we can say that the surface is emitting radiant energy with a total *energy density* u.

$$\frac{\text{Total Energy passing through aperture in 1s}}{\text{Area of aperture}} = \frac{E}{A} \tag{1}$$

Remarkably, Kirchoff showed that the energy density u depends only on the temperature of the black-body measured in ^oK and in no way upon the materials composing the black-body.

7. Stefan's Law

In 1879 J. Stefan conjectured, on the basis of extrapolation of two experimental measurements by Tyndall, that the total surface energy density u of the radiation from a black-body at a temperature T was

$$u = \sigma T^4 \tag{2}$$

where σ was a, then unknown, constant that had to be fixed by experiment. Nowadays this constant is termed the Stefan-Boltzmann constant.

$$\sigma = 5.67 \times 10^{-8} J s^{-1} m^{-2} \tag{3}$$

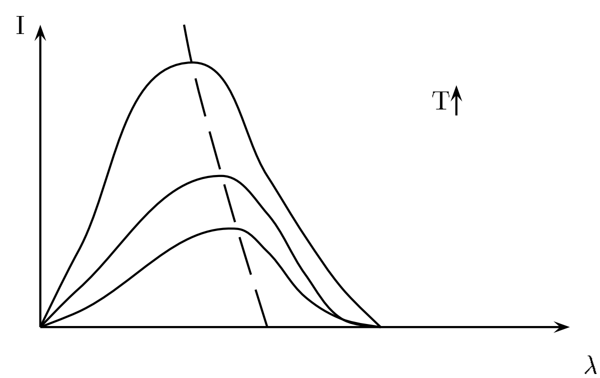
Thus a doubling of the temperature raises the surface energy density of the emitted radiation by sixteen (16) times. A theoretical derivation of Stefan's result was given in 1884 by L. Boltzmann. The fourth power law is now known as the Stefan-Boltzmann law.

8. Black-Body Radiation is Wavelength Dependent

So far we have spoken of the total energy density u without regard to the wavelength of the radiation being measured. Suppose we had a black-body emitting radiation at a temperature T^oK and pass the radiation through a filter that only allows light to pass through it if it falls in the wavelength interval λ to $\lambda + d\lambda$. We could measure the total energy passing through the filter in 1s and call it E_λ. If the surface area of the black-body is A then we would say that the energy density u_λ in the wavelength interval λ to $\lambda + d\lambda$ is

$$u_\lambda = \frac{E_\lambda}{A} \tag{4}$$

u_λ is essentially the power radiated in the wavelength interval λ to $\lambda + d\lambda$ per unit area of the black-body radiator and if λ is measured in nanometres ($1nm = 10^{-9}m$) then u_λ would be in watts per metre^2 per nanometre. So for a typical visible light of 500nm we might use a filter that passed 500 to 501nm light to determine u_λ for $\lambda = 500nm$ etc. Early workers frequently plotted the variation of u_λ with wavelength λ for a black-body at a fixed temperature T and observed results as shown below.



Notice that the black-body radiation forms a smooth *continuous* spectrum with u_λ attaining a maximum for a given temperature T. We denote the wavelength of the maxima by λ_{max}

9. Wein's Displacement Law

In 1893 Wein noted that the product of the wavelength λ_{max} with the temperature in ^oK was a constant b.

$$\lambda_{max} T = b \tag{5}$$

The constant b was later determined experimentally as

$$b = 2.897 \times 10^{-3} mK \tag{6}$$

Wein's displacement law gave an important method for determining the temperature of a black-body. Simply determine λ_{max} where the energy density attains its maximum value.

Thus for the sun we find $\lambda_{max} \sim 500nm$ leading to

$$T = \frac{b}{\lambda_{max}} = 5974K$$

Notice Wein's law is consistent with the observation that as the temperature is raised the colour of a glowing object changes from red to yellow to white, that is to shorter wavelengths.

10. Big-Bang Cosmic Radiation Background

In 1965 Penzias and Wilson measured microwave radiation intensities coming from outer space. The radiation was the same in all directions and peaked at about $\lambda_{max} = 0.107cm$. Assuming that the radiation corresponds to that of a black-body we find from Wein's law

$$T = 2.7K$$

and corresponds to relic cosmic radiation from Big Bang.

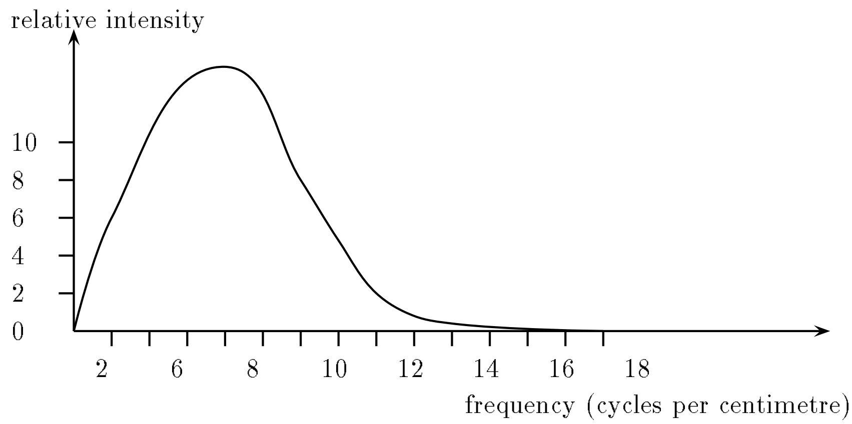
> *The most important thing accomplished by the discovery of the radiation background in 1965 was to force all of us to take seriously the idea that there was an early universe*
> – Steven Weinberg, *The First Three Minutes* New York: Basic Books (1977)

11. The Black-Body Radiation Curve

The central problem in the second half of the 1800's for physicists was to explain the form of the black-body radiation curve. Two notable results were obtained.

Wein (1896) had deduced a result that appeared to correctly represent the black-body curve for short wavelengths (e.g. in the ultraviolet) but which failed to give the correct behaviour for long wavelengths (e.g. in the infrared).

Rayleigh (June 1900) introduced a new radiation law (later to be known as the Rayleigh-Jeans law) which correctly described the long wavelength behaviour but failed catastrophically in the ultraviolet predicting the energy density rising to infinity leading to the *"ultraviolet catastrophe"*.



y 12. Planck's Solution to the Black-Body Problem

The black-body problem posed a major dilemma for physicists at the close of the 19th century. Classical physicists had had some stunning successes, most notably Newton's mechanics and Maxwell's electromagnetic theory of radiation, and yet classical physicists seemed unable to solve the black-body problem. While solutions for the long wavelength limit (the Rayleigh-Jeans law) and the short wavelength limit (the Wein law) were adequate over certain wavelenght ranges neither would cover the entire range.

A single solution that covered the entire spectral range was sought. Planck attempted desperately to produce a classical solution to the problem and successfully interpolated between the Wein and Rayleigh-Jeans laws to produce a single formula. An interpolation is *not* a derivation! In late 1900 Planck tried to assume that the radiation in a black-body cavity was produced by a set of elementary oscillators that could *continuously* radiate energy. Recall Maxwell had shown that an accelerating electric charge could emit radiation. Planck found that it was impossible to produce a solution that agreed with his interpolated result if it was assumed that the oscillators radiated continuously. He found it was necessary to assume that the energy E radiated by an oscillator working at a frequency f was radiated in discrete amounts. Indeed it appeared that the oscillators could only radiate energy in integral multiples of hf. i.e.

$$E = nhf \quad n = 1, 2, \ldots \tag{7}$$

where h is a new fundamental constant, now known as Planck's constant,

$$h = 6.626196 \times 10^{-34} Js$$

The above result became known as Planck's *quantisation postulate*. An oscillator of frequency f can only radiate energy in multiples of hf. In all previous treatments it had been assumed that the energy E of an oscillator was continuously variable whereas Planck was driven to assume the energy could only be an integral multiple of hf.

Making use of the quantisation postulate made it possible to derive a complete and seemingly exact result for the entire black-body radiation curve giving

$$u_\lambda = \frac{8\pi ch}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \tag{8}$$

where k was Boltzmann's constant

$$k = 1.380622 \times 10^{-23} JK$$

Planck's general result gave Wein's law and the Rayleigh-Jeans law as limiting special cases and furthermore lead directly to the Stefan-Boltzmann and Wein's displacement law. Stefan's constant σ and Wein's constant b became expressible in terms of fundamental constants.

Planck's quantisation postulate gave rise, all be it rather slowly, to what was to become the quatum theory of matter *and* radiation and led to the quantum technology revolution that persists today. The next major step was to be Einstein's, the subject of our next chapter.

Historical Note:- Stefan's Law

Stefan read about Tyndall's experiments in Wüllner's textbook on heat noting that Tyndall measured the total emission from a platinum wire at 1473^oK and 798^oK and reported that the emission was 11.7 times greater at the higher temperature. Stefan noted that

$$\left(\frac{1473}{798}\right)^4 \sim 11.7$$

and in 1879 Stefan concluded that for a blackbody the total radiation E is proportional to T^4.

A modern repetition of Tyndall's experiment - which was far from being a blackbody - would yield a ratio of 18.6 rather than 11.7.

Conclusion

Stefan's result was fortuitous. The theoretical proof of Stefan's Law was given by Boltzmann in 1884.

> *People have now a-days, got a strange opinion that everything should be taught by lectures. Now, I cannot see that lectures can do as much good as reading the books from which the lectures are taken. I know nothing that can be best taught by lectures, except where experiments are to be shewn. You may teach chymistry by lectures. - You might teach making of shoes by lectures!*
>
> — Samuel Johnson, February 1766 : see Boswell *Life of Johnson, London (1791)*

Chapter Seven

Particles and Waves?

> *I promise you four papers...The first..deals with radiation and energy characteristics of light and is very revolutionary...The second work is a determination of the true size of the atom from the diffusion and viscosity of dilute solutions of neutral substances. The third proves that assuming the molecular theory of heat, bodies whose dimensions are of the order of 1/1000mm, and are suspended in fluids, should experience measurable disordered motion. It is the motion of small inert particles that has been observed by physiologists, and called by them 'Brown's molecular motion'. The fourth paper exists in first draft and is an electrodynamics of moving bodies employing a modification of the doctrine of space and time; the purely kinematical part of this work will certainly interest you*
> A. Einstein, in a letter to Conrad Habicht, Spring 1905.

1. SYNOPSIS

Einstein's study of the photoelectric effect develops the concept of the photon leading to a reinterpretation of the Young double slit experiment. Photon's exhibit properties involving a wave-particle duality. Could this duality also occur for particles of matter? de Broglie makes a daring hypothesis.

2. Einstein and the Photo-Electric Effect

Planck's 1900 black-body radiation theory assumed that for electromagnetic radiation of frequency f, energy is emitted or absorbed discontinuously, in quanta of magnitude

$$E = hf \tag{1}$$

Planck's result was largely ignored until Einstein considered the subject in 1905 stating [Annalen der Physik **17**, 132 (1905)]

> *In accordance with the assumption to be considered here, the energy of a light ray spreading out from a point source is not continuously distributed over an increasing space but consists of a finite number of energy quanta which are localised at points in space, which move without dividing, and which can only be produced and absorbed as complete units.*

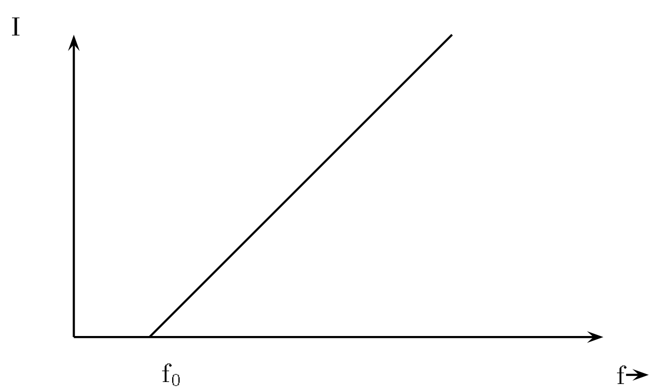
Whereas Planck considered quantised oscillators Einstein took the important step in realising that electromagnetic radiation itself was quantised.

Among Einstein's first application of the quantisation of light was an analysis of the photo-electric effect. It had been observed in the 1880's that when short wavelength light (e.g. ultraviolet) impinged on a metallic surface electrons were expelled from the surface. Light of longer wavelengths failed to expel any electrons, i.e. there was a sharp wavelength cutoff. The photo-electric effect appeared to be inconsistent with classical electromagnetic theory.

Einstein's resolution of the photo-electric effect proceeded from the assumption that light consisted of quantised photons. Einstein wrote further in his 1905 paper

> *According to the concept that the incident light consists of energy quanta (photons) of magnitude hf,... one can conceive of the ejection of electrons by light in the following way. Energy quanta penetrate into the surface layer of the body (the target electrode), and their energy is transformed, at least in part, into kinetic energy of electrons. The simplest way to imagine this is that a light quantum delivers its entire energy to a single electron; we shall assume that this is what happens. The possibility should not be excluded, however, that electrons might receive their energy only in part from the light quanta. An electron to which kinetic energy has been imparted in the interior of the body (the target electrode) will have lost some of its energy by the time it reaches the surface. Furthermore, we shall assume that in leaving the body each electron must perform an amount of work P characteristic of the substance. The ejected electrons leaving the body with the largest normal velocity will be those that were (located exactly on) the surface. The kinetic energy of such electrons is given by $hf - P$.*
>
> *If the body is charged to a positive potential V_0 and is surrounded by conductors at zero potential, and if V_0 is just large enough to prevent loss of electricity, it follows that*
>
> $$V_0 e = hf - P,$$
>
> *where e denotes the electronic charge ...*

The above equation became known as Einstein's photo-electric equation and led to an experimental determination of the ratio h/e and since e was known it was possible to use the photo-electric effect as a means of determining Planck's constant h.



3. Einstein's Photons

Einstein's photons travel at the speed of light c and as a result of Einstein's relativity theory can *never* be found, in vacuum, at a speed less than c. There is no situation where the photon could be found at rest. It either travels at the speed c or it does not exist. A further consequent of relativity is that the photon posseses no *rest* mass. At the instant of its creation it takes off at the speed c. If it is absorbed by matter it ceases to exist as a photon. The photon carries with it *momentum* p

$$p = \frac{hf}{c} = \frac{h}{\lambda} \tag{2}$$

This means that a photon impinging on a surface will exert on that surface a force and hence there exists a *radiation pressure* that can be experimentally measured.

4. How numerous are photons?

A 100 watt light bulb emits approximately 1 watt of visible light at about 500nm. How many photons of visible light are emitted each second?

The energy of one photon of light at 500nm is

$$E = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{500 \times 10^{-19}}$$
$$= 4 \times 10^{-19} J$$

One watt $= 1Js^{-1}$ and thus the number of photons emitted in 1s is

$$= \frac{1}{4 \times 10^{-19}} = 2.5 \times 10^{18}$$

which is an enormous number.

5. Sensitivity of the human eye

The human eye can detect light down to a few photons/second. Suppose we place a source emitting 10^{18} photons/second at the centre of a large sphere of radius R metres and suppose the photons are emitted uniformly in all directions. The number of photons N striking a square metre of the surface of the sphere will be equal to

$$N = \frac{10^{18}}{4\pi R^2} \tag{3}$$

Under favourable conditions the human eye has a receiving diameter of $\sim 6mm$ and hence an area of $\sim 3 \times 10^{-5} m^2$. Thus the number of photons the eye receives from the source at a deistance R will be

$$n = 3 \times 10^{-5} N = \frac{7.8 \times 10^{11}}{R^2} \tag{4}$$

Suppose we ask that $n = 10$ be the number of photons entering the eye/second at the distance R then

$$R^2 = 7.8 \times 10^{10}$$

and hence

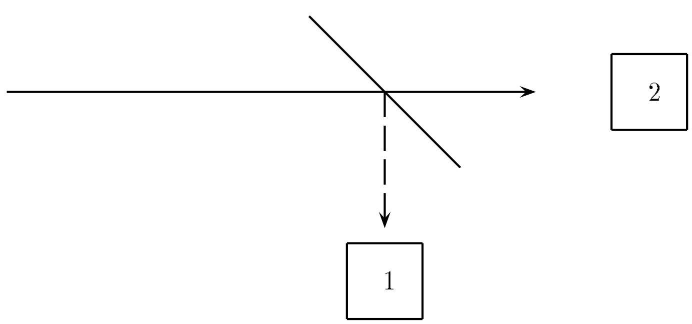
$$R = 2.8 \times 10^5 m = 280km$$

Q1 Discuss the assumptions made in the above calculation.

6. Photons can not be split

For two excellent accounts of this topic see the first chapter of *Principles of Quantum Mechanics* by P.A.M. Dirac (that chapter is readable by any physics student - the succeeding chapters are demanding!) and *Take a Photon ...* by O.R. Frisch in Contemporary Physics **7** (1985).

Einstein assumed that a photon could not be divided into fractions of a photon. Consider an arrangement as below



Can we use a beam splitter to split a photon? Suppose we adjust the photocells so that they count only if photons with energy between $\frac{hc}{\lambda}$ and $\frac{2}{3}\frac{hc}{\lambda}$ will be counted. We can arrange for the beam splitter to divide the beams of photons into a transmitted beam and a reflected beam of equal intensity. In that case both photocells will count at the same rate.

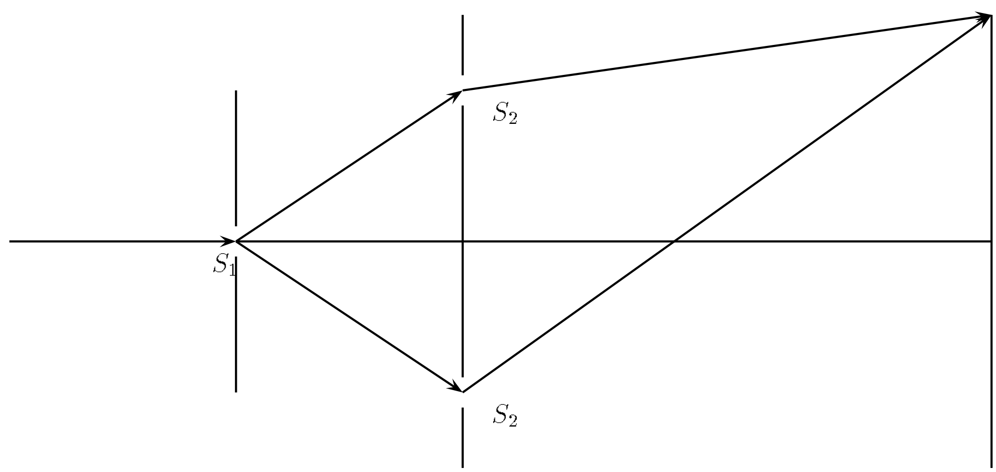
On the classical wave theory we would expect the wave train to be divided and neither counter to click. Experimentally we find that the wavelength of the light in the two beams is unchanged and the counting rate registered by photocell 2 is halved from what it would be if the beam splitter were absent.

From the above observations we may conclude that photons do not split.

Now lower the intensity of the lamp until photons arrive singly. We now find that as each photon strikes the beam splitter it either passes through and photocell 2 registers a count or it is reflected and photocell 1 registers a count. If we carry out observations over a long period of time we will find that the two photocells register essentially the same number of counts. However, for an *individual photon* we can make no deterministic prediction as to which counter will be activated.

7. Young's Double Slit Experiment (Again)

Let us return to Young's double slit experiment that gave rise to the revival of the wave theory of light.



Let us consider photons coming from the source. We know experimentally that an interference pattern of bright and dark bands is produced. If either slit is blocked no pattern is seen. If both slits are open the interference pattern is seen. Suppose we regard the beam of light as split into two beams of photons of equal intensity, i.e. equal numbers of photons. If the two components interfere destructively we would require a photon in one beam to annihilate a photon in the other to produce a dark spot or two photons to somehow combine together to produce four photons. Either situation would violate conservation of energy. Evidently photons do not arrive at the spots where there are dark fringes. To overcome this paradox it is necessary to assume that the photon is partly present in both beams. Each photon only interferes with itself. No interference occurs between two different photons.

We can again make no meaningful statement that a particular photon is in a particular beam. Its position between the slits is indeterminate. Note an interference pattern is expected, and found !, even when the intensity of the source is so low that photons reach the slit system one at a time. Photons exhibit a full range of phenomena associated classically with waves (e.g. diffraction and interference effects) as well as particle-like properties as shown in the photo-electric effect.

8. Do Electrons exhibit wave-like properties?

In 1923 Prince Louis de Broglie suggested that atomic particles might exhibit a wave-like aspect to their behaviour. He had noted that photons carried a momentum p that was related to the energy E of the photon by

$$E = pc \tag{5}$$

Noting Planck's quantisation result

$$E = h\nu = \frac{hc}{\lambda} \tag{6}$$

he could write for a photon

$$p = \frac{h\nu}{c} = \frac{h}{\lambda} \tag{7}$$

and hence

$$\boxed{\boxed{\lambda = \frac{h}{p}}} \tag{8}$$

de Broglie then made the daring jump by suggesting that a particle of mass m travelling with a speed v would exhibit wave-like properties with a wavelength

$$\boxed{\boxed{\lambda = \frac{h}{mv}}} \tag{9}$$

The wavelength so calculated is known as the de Broglie wavelength. Somewhat remarkably de Broglie presented his results in French in his thesis and in English in his scientific paper on the subject.

L. V. de Broglie, *A tentative theory of light quanta*, Phil. Mag. (London) **47**, 446 (1924).

9. Examples

1. A 1000kg car travelling at $10ms^{-1}$ ($\sim 36kph$)

$$\lambda = \frac{6.6 \times 10^{-34}}{1000 \times 10} = 6.6 \times 10^{-38} m$$

2. A 10gm bullet travelling at $500ms^{-1}$

$$\lambda = 1.3 \times 10^{-34} m$$

3. An electron with a kinetic energy of $1eV$

$$\lambda = 1.2 \times 10^{-9} m = 1.2nm$$

4. For thermal neutrons $\frac{1}{2}mv^2 = \frac{3}{2}kT$ and hence $mv = \sqrt{3mkT}$ leading to

$$\lambda = \frac{h}{\sqrt{3mkT}}$$

For T = 300K we obtain $\lambda \sim 15nm$

The first two examples involve wavelengths that are *very* small compared with the atomic dimensions ($\sim 0.1nm$). The third example involving electrons results in a wavelength comparable with atomic dimensions and hence could be expected to give rise to interference and diffraction phenomena at the atomic scale. N.B. For more energetic electrons it is necessary to replace the classical momentum (mv) in Eq.(9) by its relativistic momentum. Likewise in we expect similar interference and diffraction effects for neutrons. The 1994 Nobel Prize in Physics was for the practical development of neutron diffraction techniques and then application to studies of solids and liquids. Neutrons carry no nett charge so are unaffected by Coulomb fields but have a non-zero magnetic moment so are sensitive to magnetic fields and electron spins and can thus supply information not obtainable from X-ray studies.

Q2 Protons have a wavelength also comparable with that of the neutron and are more easily prepared as beams why are they not also used in atomic structure studies?

de Broglie's matter theory was supported in 1927 by the experimental work of Davisson, Germer and Thomson. Later experimental work was to show diffraction and interference effects for beams of neutrons, helium atoms and hydrogen molecules. We can conclude that *all matter exhibits wave-like properties* These properties become discernible at the atomic scale.

10. "Seeing" an electron

Could we "see" an electron? To "see" an electron implies that we scatter light off it and detect the scattered or reflected light. Ordinary visible light will not suffice as the size of the electron is very much less than the wavelength of visible light. This suggests we should use very short wavelength radiation such as γ rays. However the effect of such energetic radiation striking the electron will be to impart a velocity to the electron. If the position of the electron is to be determined accurately shorter wavelength

, more energetic radiation must be used. Thence the disturbance of the electron's motion and hence the electron's momentum. The more accurately the position of the electron is determined the greater is the inaccuracy in the momentum of the electron.

11. Heisenberg's Uncertainty Principle

In 1926 Werner Heisenberg (1901-76) examined the problem of simultaneously determining the position and velocity of an electron using a hypothetical $\gamma-$ray microscope. He found that every process of measurement disturbs the measured object no matter what the process and no matter what the object. This disturbance causes uncertainties or inaccuracies in the position and velocity of the object. Heisenberg's result maybe stated as: we cannot *simultaneously* measure the position and velocity of an object to arbitrary precision.

The product of the uncertainty in position (Δx) with the uncertainty in the $x-$component of the velocity (Δv_x) cannot be reduced to zero

$$\Delta x \times \Delta v_x \approx \frac{h}{2\pi m} \approx \frac{10^{-34}}{m}$$

Heisenbergs result is often written in terms of the momentum $p = mv$ for a particle as

$$\Delta x \times \Delta p_x \approx 10^{-34}$$

In the case of a photon the momentum is *precisely* $p = \frac{hf}{c}$ and as a consequence its position is completely indeterminate - it cannot be localised in space at all.

N.B. An important consequence of Heisenberg's work is that a particle such as an electron cannot be associated with a well-defined path through spacetime. The concept of a *trajectory* implies that, at a given instant, one may specify where a particle is *and* the direction and speed with which it is moving. The concept of a well defined trajectory is incompatible with the requirements of quantum theory. If the position of an electron at one time is A and another time B we cannot trace out a well-defined path of motion connecting A and B.

12. Matter Waves and Interference (Again)

As noted earlier a photon cannot be localised at a place. The interaction of a photon with a detector occurs at a definite place and time. Prior to this interaction the photon has no meaningful (or determinate) position. This point needs to be clearly realised in analysing Young's double slit experiment.

Likewise in the case of electrons passing through a double slit system we cannot determine the electron trajectories or paths with certainty. The electron is believed to be a point-like entity without structure. The precise location of the point may not be well-defined. In that sense the electron is not a wave but the way it moves about is controlled by wave-like principles.

Matter waves may be viewed as *probability* waves which tell us where the particle is most likely to be found. The electron interference pattern is built up of a series of electron-detector interactions. Observing a large number of such interactions builds up the interference pattern. No predictions can be given of the actual path taken by an individual electron before interacting with the detector. We cannot determine which slit the electron passes through without destroying the interference phenomena itself.

> *We believe it is an appropriate time to review the evidence, and to see if the case for a critical density Universe is compelling. Inter alia the purpose of so doing is to emphasize that this is indeed an experimental question, where theory - no matter how dear it may be to us - will eventually have to bow to the experimental evidence*
>
> — P. Coles and G. Ellis *The case for an open Universe*, Nature **370**, 609 (1994)

Chapter Eight

From Line Spectra to Particle Physics

One of the most striking phenomena which have been observed in this experiment is the occasional simultaneous appearance of paired tracks consisting of one positive particle and one negative with a common point of origin.

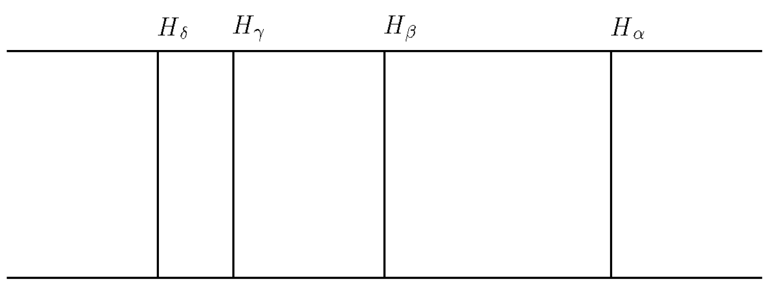
— C. D. Anderson and S. H. Neddermeyer, Phys. Rev. **33**, 1034 (1933)

1. SYNOPSIS

The development of quantum theory, after Planck's initial black-body studies, was greatly influenced by the experimental studies of the line spectra of the elements, with Balmer's observations concerning the simple line spectra of the hydrogen atom leading up to Bohr's semiclassical quantum model of the $H-$atom. We then switch to questions relating to the discovery of antimatter and its relationship to particle physics and the ultimate constituents of matter.

2. The Hydrogen Atomic Spectrum

The simplest spectrum is that of the hydrogen atom which involves relatively few lines in the visible.



with the following lines being prominent

line	wavelength λ nm
H_α	656.2
H_β	486.1
H_γ	434.0
H_δ	410.1

3. The Balmer Series

In 1884 the Swiss school teacher, J. Balmer, learnt of the existence of the H spectral lines H_α, H_β and H_δ and noted the ratios

$$H_\alpha/H_\beta = \frac{656.2}{486.1} = 1.3499 \approx \frac{27}{20} = 1.35$$
$$H_\alpha/H_\delta = \frac{656.2}{410.1} = 1.600 \approx \frac{8}{5} = 1.6$$
$$H_\beta/H_\delta = \frac{486.1}{410.1} = 1.1853 \approx \frac{32}{27} = 1.1852$$

Balmer then conjectured that the wavelength of any member of the series would be given by

$$\lambda_n = \frac{n^2}{n^2 - 2^2}\lambda_0 \quad \text{where } n = 3, 4, 5\ldots$$

with $\lambda_0 = 364.56nm$. Using Balmer's conjecture we find

n	λ_n			
3	$\frac{9}{9-4}\lambda_0$	$= \frac{9}{5}\lambda_0$	$= 656.2nm$	H_α
4	$\frac{16}{16-4}\lambda_0$	$= \frac{4}{3}$	$= 486.1nm$	H_β
5	$\frac{25}{25-4}\lambda_0$	$= \frac{25}{21}\lambda_0$	$= 434.0nm$	H_γ
6	$\frac{36}{36-4}\lambda_0$	$= \frac{9}{8}\lambda_0$	$= 410.1nm$	H_δ
∞	λ_0		$= 346.6nm$	H_∞

His conjecture readily reproduced the observed ratios and correctly gave the H_γ line. Balmer's result, having absolutely no theoretical foundation, was later to play a key role in Bohr's quantum model of the $H-$atom and in the subsequent development of Schrödinger's equation.

4. Discovery of the Positron

Rutherford showed that the atom consisted of a core of protons (later also neutrons) surrounded by a number of electrons. For a neutral atom the number of electrons was equal to the number of protons in the nucleus. Such a model immediately ran into serious conflict with classical electromagnetism. If the electron is a classical charged particle in orbit about the nucleus then it would be under constant acceleration and should radiate, losing energy and eventually collapse into the nucleus in about $10^{-9}s$!

In 1913 Niels Bokr (1885-1962) made a semiclassical derivation of the energy levels of the hydrogen atom. He essentially extended Planck's quantization postulate to say that the angular momentum of an electron would also be quantized and used a mixture of classical Newtonian mechanics and the classical Coulomb's law with the quantization postulate and obtained a correct expression for the energy levels of a hydrogen atom. Bohr had assumed that the energy states of his quantised version of Rutherford's planetary atom involved a lowest energy state, the *ground state*, from which it did not radiate.

In 1926 Schrödinger tried to introduce a relativistically correct equation for the hydrogen atom but he found it gave an inadequate description of the fine structure of the hydrogen atom and abandoned his equation (later to become known as the Klein- Gordon equation, a relativistically correct equation for describing bosons which we shall meet later). He then wrote the equation nowadays universally known as the *Schrödinger equation*. His equation, while relativistically unsatisfactory, has to this day been the principal equation of low energy quantum theory. Schrödinger's equation had the considerable advantage over the crude Bohr model in that it led directly to the possibility of calculating properties other than just energy, such as transition probabilities and was also capable of extension to many-electron systems.

In 1928 P. A. M. Dirac produced a new equation to describe the properties of an electron taking into account both quantum theory *and* relativity in a consistent manner. Dirac's new equation was an embarrassment as it possessed four solutions instead of the two expected for the two spin states of the negatively charged electron. These extra solutions, as realised by Dirac, seemed to be appropriate to the properties of a positively charged particle of the same mass as the electron. The only known positively charged particle was the proton some 1836 times the mass of the electron. J. R. Oppenheimer showed that Dirac's extra solutions could not be associated with the proton.

In 1932 C. D. Anderson and S. Neddermeyer studied the properties of cosmic rays interacting with matter using a cloud chamber and on 2 August 1932 obtained a remarkable photograph that could only be identified with a *positively* charged particle with the same mass as the electron! This particle, the *positron*, was indeed the particle described by Dirac's extra solutions and was the *antiparticle* of the electron.

5. Antimatter

The discovery of the positron heralded the discovery of a new form of matter known as *antimatter*. Dirac's theory, along with the Klein-Gordon equation rejected by Schrödinger, predicted that for every particle there should be a corresponding antiparticle of the same mass with the opposite sign of the electric charge and magnetic moment. Indeed it was predicted that even neutral particles such as the neutron, or more exotically the neutrino, would have corresponding antiparticles. For some neutral particles (bosons) the particle and antiparticle would be equivalent as is the case for the photon.

In 1955 it became possible to produce in the laboratory *antiprotons* and *antineutrons*. With an antiproton and a positron it becomes possible to create antihydrogen and in principle, though not in practice, it should be possible to create the antiparticle equivalent of any object.

6. Matter-Antimatter creation and annihilation

$\gamma-$rays are essentially particles of light or *photons* of high energy and carry no electric charge. They are in essence packets of electromagnetic energy. If the energy of a $\gamma-$ray (E_γ) is greater than the rest energy of an electron and a positron it is possible for an electron-positron pair to materialise

$$E_\gamma > 2m_e c^2 = 1.022MeV \tag{1}$$

Conversely an electron and a positron may come together and annihilate each other to for a pair of $\gamma-$rays (at least two γs are required to conserve linear momentum.

$$e^+ + e^- \rightarrow 2\gamma \tag{2}$$

Q1. Use the above two equations to show how energetic $\gamma-$rays can be reduced to $\gamma-$rays of energy $\sim 0.511MeV$.

In general matter brought into contact with antimatter will annihilate producing neutrinos, antineutrinos, $\gamma-$rays etc.

Q2. Matter and antimatter appears to be completely symmetrical and we would at first sight expect a universe with equal quantities of matter and antimatter. In practice the universe appears to be made of only one form of matter. Why?

7. Neutrinos and Antineutrinos

Early studies of $\beta-$decay created a crisis in physics. It appeared to violate both the conservation of energy and the conservation of spin statistics. Pauli realised in 1930 that both conservation laws woulsd be *saved* by hypothesising the existence of a particle that had the same spin as the electron ($s = \frac{1}{2\hbar}$) but necessarily carrying no charge. He assumed that the particle had only very weak interactions with matter. The particle was called the neutrino ν with its corresponding antiparticle being an *antineutrino* $\bar{\nu}$. Direct observation of the neutrino was made by Cowan and Reines in 1953.

Neutrinos are produced in enormous numbers in nuclear reactors and in the sun. Approximately 10^{11} pass through a given square centimetre of the earth's surface every second. The neutrino is one of the most abundant particles in the universe and has a mass close to zero if not zero itself.

Q3. Supernova 1987a was only visible in the Southern hemisphere. Japanese researchers detected a few neutrinos from 1987a. Why were they able to detect them?

8. Quarks and Subnuclear Structure

Three quarks for Muster Mark, Finnegan's Wake, James Joyce

Are the neutron and proton elementary entities or do they have a substructure? Might not the nucleons be composites of some other particles? If high energy electrons are scattered off nucleons it appears that nucleons do indeed have a substructure and consist of parts (*partons*). It is now believed that nucleons are made up of two types of *quarks* - $u-$ and $d-$quarks with electric charges

$$q_u = \frac{2}{3}e \qquad q_d = -\frac{1}{3}e$$

The proton involves two u quarks and one d quark while the neutron involves one u quark and two d quarks where e is the magnitude of the electron charge.

$$proton \quad uud \quad Q_p = \left(\frac{2}{3} + \frac{2}{3} - \frac{1}{3}\right) e = e$$

$$neutron \quad udd \quad Q_n = \left(\frac{2}{3} - \frac{1}{3} - \frac{1}{3}\right) e = 0$$

The antiproton and antineutron are built out of *antiquarks*

$$\bar{u} \quad q_{\bar{u}} = -\frac{2}{3}e \qquad \bar{d} \quad q_{\bar{d}} = \frac{1}{3}e$$

$$antiproton \quad \bar{u}\bar{u}\bar{d} \quad Q_{\bar{p}} = -e$$

$$antineutron \quad \bar{u}\bar{d}\bar{d} \quad Q_{\bar{n}} = 0$$

Notice that the neutron and antineutron while electrically neutral involve quarks that carry an electric charge. This makes it possible for the neutron and antineutron to exhibit magnetic properties even though they are electrically neutral.

$\beta-$decay can be regarded as the decay of a neutron in the nucleus via

$$n \longrightarrow p + e^- + \bar{\nu}$$

or in terms of their quark constituents

$$udd \longrightarrow uud + e^- + \bar{\nu}$$

and hence

$$d \longrightarrow u + e^- + \bar{\nu}$$

9. The Strong Nuclear Force

We have already met the very weak gravitational force via Newton's law of gravitational attraction and the vastly stronger electromagnetic forces as seen for example in the force law of Coulomb for electric charges. Neither of these forces is strong enough to account for the observed strong forces that bind nucleons together to produce stable nuclei. The strong nuclear force may be studied by observing the way it scatters particles that come close to the nucleus. Beams of energetic protons or neutrons may be made to scatter off nuclei in a controlled manner. Among the first surprising result was the observation that the strong nuclear force seemed to be essentially independent of whether neutrons or protons were used. That is, the nuclear force seemed to be *charge independent* the forces being

$$p - p \approx p - n \approx n - n$$

We have already noted that the mass of the neutron and proton are same to approximately one part in a thousand. This observation lead Heisenberg to suggest that the two particles could be viewed as different charge states of a single particle, the *nucleon*. This suggested that if one could somehow switch off the Coulomb force leaving just the strong nuclear force these two particles would have exactly the same mass. These observations were strongly analogous to the observation of electron spin in atoms. For a single electron the two possible spin states have the same energy and are said to be *two-fold degenerate*. This two-fold degeneracy may be lifted by application of an external magnetic field to produce two states of slightly different energy. This to physicists is an example of *broken symmetry*. To explore these ideas we need first to remark on the subject of symmetry in physics but this must await the next chapter.

The contemporary scientific revolution has effected the dissolution of one of the most extensive superstitious beliefs of the age: the materialistic, clockwork universe of nineteenth-century physics. But perhaps all of this need not be considered on the old true/false scale of dualities and polarities. Perhaps it can be used merely to suspend temproarily our disbeliefs

— Sara Maitland *Women fly when men aren't watching*, Virago Press, London (1993)

Chapter Nine

Ideas of Symmetry in Physics

Deep down inside, no-one understands quantum mechanics ...

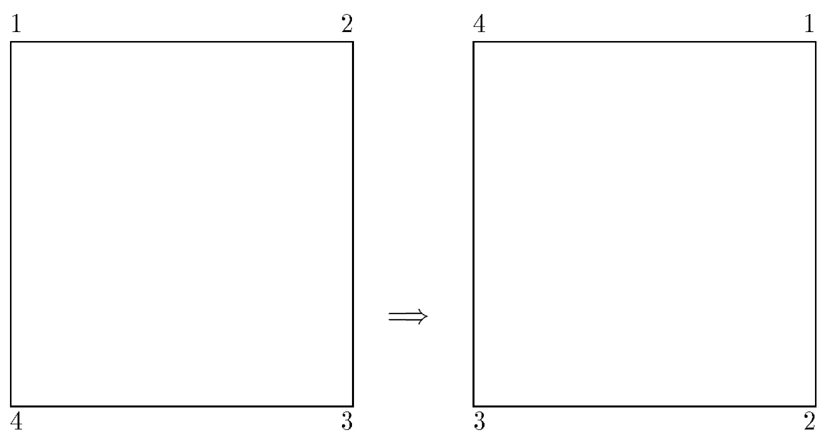
— Murray Gell-Mann, *Lecture at University of Canterbury, Christchurch, New Zealand (1978)*

1. SYNOPSIS

Symmetry plays a key role in modern physics. Here we give a number of examples of symmetries and outline the various types of symmetries encountered in physics. We comment on the relationship of symmetry to the conservation laws of physics and the significance of the breaking of symmetries. Finally we give some practical demonstrations of the distinction between rotations through 2π and 4π. In a later chapter we will relate some of these ideas to the understanding of the various particles of physics.

2. Examples of Symmetry

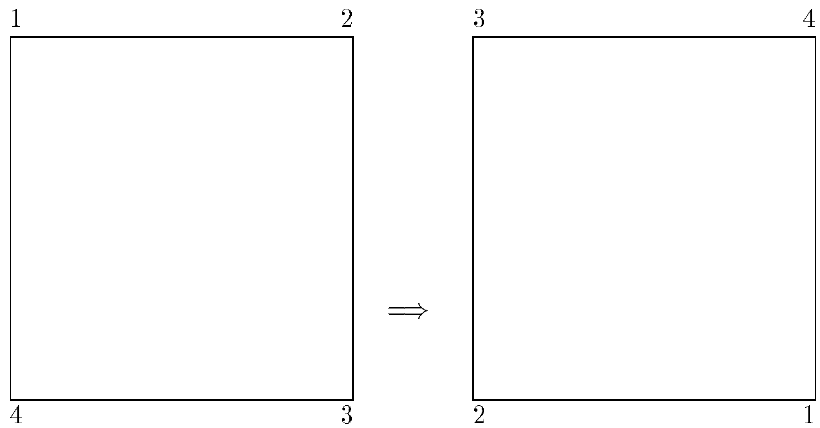
Symmetry and invariance are closely related. Symmetries are usually associated with an operation on a system that transforms it into itself in such a manner that the system after the transformation is indistinguishable from its initial state. If the symmetry is perfect, which is rarely the case, then it should be experimentally impossible to distinguish any change in the system after carrying out the symmetry transformation. A simple example is the case of a square lying on a plane surface. If the square is rotated through an angle of 90^o about its centre it should be indistinguishable from the original unrotated square. If that is the case then the square is said to be symmetric with respect to a rotation through a finite angle of 90^o about its centre. The properties of the square are said to be *invariant* with respect to such a rotation. In picturing such a transformation it is useful to attach the integers $1, 2, 3, 4$ to the vertices of the original square and to display the rotated square displaced from the original square as below



rotation

Note that our rotation could be regarded as equivalent to a permutation of the vertices of the square such that $1 \rightarrow 2, 2 \rightarrow 3, 3 \rightarrow 4, 4 \rightarrow 1$. It is not difficult to see that there are eight distinct permutations that leave our square invariant and each of these permutations can be associated with either a rotation about the centre of the square or a reflection about the diagonals or bisectors of the square. This gives an example of a *finite symmetry* characterized by a finite number of symmetry operations which form the elements of a *finite group*.

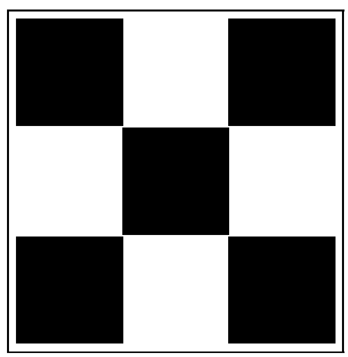
Our square is also symmetric with respect to an inversion through its centre. In that case the inversion symmetry is equivalent to a rotation through 180^o about the centre as seen below



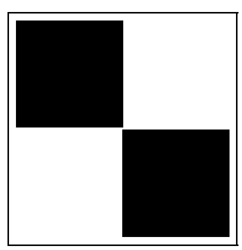
inversion

The Platonic solids, the cube, octahedron, dodecahedron and icosahedron all posssess a centre of inversion which, however, can not be made equivalent to any set of rotations.

We could decorate our square and still leave a square that has the full symmetry of the plain square as shown, for example, below

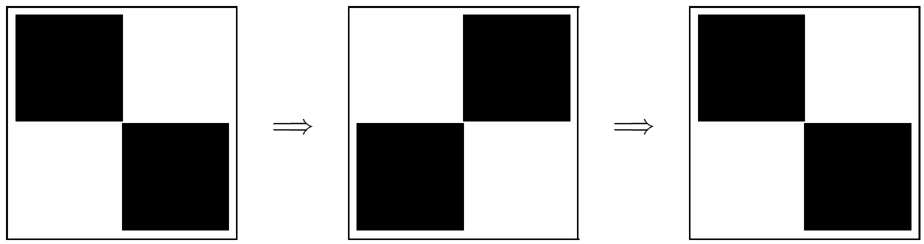


The figure below clearly no longer possesses the symmetry of the plain square as clearly it does not go into itself under a simple rotation of 90^o about its centre.



However, we could extend our symmetry by introducing a more complicated transformation - first carry out the rotation and then a counterchange operation that turns black into white and white into black.

This two step process is illustrated below.



rotation **counterchange**

This is an example of a black and white symmetry beautifully outlined in four articles published by H.J. Woods in the British Journal of the Textile Institute in the early 1930's, the counterchange operation arising naturally in the production of textiles. These Black and White groups are commonly referred to as Shubnikov groups though it is clear that Woods' work preceded that of Shubnikov. It was Landau who supplied the interpretation in the physics of magnetism by regarding the counterchange operation as the equivalent of flipping a spin.

Permutational symmetry is important in considering the interchange of identical objects. A diatomic molecule with each atom being of the same isotope will exhibit permutational symmetry - interchange of the two atoms leaves the molecule in position indistinguishable from its former position. If the two atoms involve different isotopes then the permutational invariance is broken.

The preceding examples all involve finite symmetry transformations. Other examples can involve *continuous transformations*. Thus a blank coin will exhibit cylindrical symmetry with respect to any rotation about an axis perpendicular to its centre. A sphere devoid of any markings and perfectly regular may be rotated into itself by any rotation about any axis that passes through its centre. An atom sitting in free space exhibits spherical symmetry. Since there is no preferred direction in space there is no preferred direction to align the angular momentum of the atom with the result that the $2J+1$ states $|JM\rangle$ are degenerate. Break the spherical symmetry by placing the atom in a magnetic field which destroys the spherical symmetry locally and the degeneracy is lifted as in the Zeeman effect.

■ 3. Continuous and Discrete Symmetries

The above examples of symmetries may be divided into two classes, *continuous* and *discrete*. Discrete symmetries such as reflections, inversions, permutations and finite rotations are associated with *multiplicative quantum numbers* whereas continuous symmetries are associated with *additive quantum numbers* such as, for example, angular momentum addition.

■ 4. Symmetry, Conservation Laws and Impossible Experiments

The existence of a symmetry is always tentative and experiments are required to determine the limits of applicability of a given symmetry. No symmetry can be considered as a perfect symmetry. The object of much of fundamental physics is the establishment of the limits of particular symmetries and where a symmetry is broken to explain the nature of the symmetry breaking mechanism. One cannot overemphasize the connection between symmetry and experiment. In 1905 Emmy Nöether made the remarkable observation the conservation laws in physics are associated with particular symmetries. Thus conservation of linear momentum was associated with invariance with respect to spatial translations, angular momentum with spatial rotations etc. Parity conservation was associated with the equivalence of the mirror image of an interaction and the real interaction. Every symmetry can be considered as a statement that a certain experiment is impossible. If the experiment is possible then the symmetry must at least be broken. Thus for the parity conservation the impossible experiment would be to detect a difference between the mirror image of a real process and the process itself. If you could detect such a difference then the symmetry is broken and parity is not conserved. Indeed, Madame Wu succeeded in 1956 in making a $\beta-$decay experiment that showed an asymmetry with respect to parity and hence parity conservation was broken by weak interactions. Still more subtle was the demonstration, in 1964 by Fitch and Cronin, of CP violation for K mesons .

■ 5. Integer and Half-Integer Angular Momenta

The basic particles of the universe can be divided into two distinct classes

> **Fermions** which have half-integer spin such as the electron, nucleon, neutrinos quarks etc all of which follow Fermi-Dirac statistics and involve antisymmetric states,
>
> **Bosons** which have integer spin such as the mesons, photon, gluons, graviton etc all of which follow Bose-Einstein statistics and involve symmetric states.

Bosons and fermions behave differently under particle interchange or a rotation through 2π.

■ 6. Examples of 2π and 4π Rotations

Our intutitive expectation is that if we rotate a system through 2π or if we circumnavigate a system once we will return to the initial state. I now give you three demonstrations where the niave expectation does not hold.

The Möbius Strip

We can readily make ourselves a Möbius strip by taking a longish narrow strip of paper and rotating one end through 180^o and then bringing the two ends together and sticking them together with glue. Now place a reference mark on the strip and from that mark draw a line along the middle of the strip continuing until you return to the reference mark. You will note that in doing this you have traversed the strip *twice!*.

Cup and Saucer

Place a cup on a saucer and hold it in the palm of your hand. Now turn the cup and saucer by rotating your hand through 2π. This leaves your hand twisted. To return to the original untwisted configuration rotate through a further 2π. To do that you will need to move your hand over your head to complete the total rotation through 4π and return to the original position. This is more dramatic if the cup is partially filled with water - this makes the cup more stable though students are likely to find the failure of the experiment more memorable.

Rotation of a Triangle

Make an equilateral triangle with distinguishable sides. Make a hole in each vertex and attach to each hole a differently coloured tape, e.g. red, green and blue. Attach the loose ends to fixed points. Now rotate the triangle through 2π by turning it over twice so as to develop a twist in two of the tapes. At this stage it is impossible to undo the twist without reversing the rotation or cutting the tapes. Now rotate the triangle through a further 2π so that the two twisted tapes are further twisted. I now assert that the twist can be removed while keeping the triangle in a fixed position and not untying any of the tapes. Indeed the twists incurred by any rotation through an even multiple of 2π may be undone but not for odd multiples of 2π.

■ 7. Symmetry and Selection Rules

The existence of a symmetry usually implies that certain processes are not possible. If they were possible then the symmetry would be broken. Part of the application of symmetry considerations is to determine selection rules and to determine the conditions under which these selection rules are broken. Thus in the case of transition matrix elements those that satisfy the selection rules are said to correspond to *allowed* transitions and those that violate the selection rules are termed *forbidden* transitions. In the case of electric dipole transitions in atoms there are the well-known angular momentum selection rules

$$\Delta S = 0 \quad \Delta L = 0, \pm 1 \quad \Delta J = 0, \pm 1$$

and in each case $NOT\ 0 \leftrightarrow 0$. These selection rules arise directly from the fact that the electric dipole operator is spin-independent and transforms like a vector with repect to the group of angular rotations SO_3. The electric dipole operator has *odd* parity so can only connect states of opposite parity.

As a consequence of the above selection rules we would expect electric dipole transitions involving ${}^1S_0 \leftrightarrow {}^3P_0$ to be strongly forbidden. Nevertheless such transitions are observed in gaseous nebulae. The selection rules on the quantum numbers S and L can be broken by the spin-orbit interaction but it cannot break the $NOT\ J = 0 \leftrightarrow J = 0$ selection rule.

■ 8. Resolution of the ${}^1S_0 \leftrightarrow {}^3P_0$ Puzzle

To understand the origin of these seemingly highly forbidden transitions we need to first ask "What is the total angular momentum of an atom?". The quantum number J represents the total *electronic* angular momentum of the atom. But the atom has a nucleus that also has a total *nuclear* angular momentum I so that the total angular momentum of the atom F is

$$\mathbf{F} = \mathbf{I} + \mathbf{J}$$

I will be an integer (or half-integer) if the number of protons plus neutrons is *even* (or *odd*) while J will be an integer (or half-integer) if the number of electrons is *even* (or *odd*). If J is an integer and I is a half-integer then F will be a half-integer. If $I \geq \frac{1}{2}$ then the nucleus may possess a nuclear magnetic moment that can couple to the electronic moments leading to a mixing of states of different electronic angular momentum J leading to a breakdown of the $\Delta J = 0$ NOT $0 \leftrightarrow 0$ selection rule as occurs for example in the transition array associated with the $3s^2 \leftrightarrow 3s3p$ transition array.

■ 9. Concluding Remark

Symmetry provides a very useful method of organising information in physics as we shall see in the next chapter when we try to organise the particles of nature into recognisable patterns.

> *Compared to this discovery, the theory of relativity, which you all understand to be a great revolution in physics, was only a minor modification... You think I'm going to explain it to you so you can understand it? No, you're not going to be able to understand it ... You see my physics students don't understand it either. That is because I don't understand it. Nobody does. I'm going to describe to you how Nature is, and if you don't like it, that's going to get in the way of your understanding it. It's a problem that physicists have learned to deal with: they've learned to realise that whether they like a theory or not is not the essential question. Rather, it is whether or not the theory gives predictions that agree with experiment ... The theory of quantum electrodynamics describes Nature as absurd from the point of view of common sense. And it agrees fully with experiment... I'm going to have fun telling you about this absurdity, because I find it delightful. Please don't turn yourself off because you can't believe that Nature is so strange...*
> Richard Feyman, *QED: the strange theory of light and matter*, A series of lectures first given in New Zealand, later published as a book and recently translated into Polish. Available in the Physics Library.

Chapter Ten

Symmetry and the Particle Zoo

THE CERN COUNCIL HAS APPROVED THE CONSTRUCTION OF THE LARGE HADRON COLLIDER (LHC). The huge proton- proton collider, to be housed in the existing 27-km tunnel used by the Large Electron Positron collider, will be built in two stages. First, 10-TeV beams will be achieved by about the year 2004. Head-on collisions would have a total energy of 20 TeV. Primary topics of study in this first phase would be top quarks and CP violation. In the second phase, to start in 2008, the total collision energy would be boosted to 28 TeV. At this energy the search for the Higgs boson would be a high priority.

1. SYNOPSIS

In the chapters remaining for this semester I shall be discussing aspects of our current understanding of the forces of nature and what can be expected from the construction of the large hadron collider in Geneva. We will first extend our earlier remarks on symmetry, with some deliberate repetition, and its consequences and trace the development of our knowledge of the particles that make up matter as we know it.

■ 2. Why Symmetry?

Symmetry is usually associated with an action or transformation of a system or object such that after carrying out the operation the system or object is in a state indistinguishable from that which it had prior to carrying out the action or transformation. Thus there is a close relationship between symmetry and impossible experiments. The existence of a symmetry implies that it is impossible to devise an experiment to distinguish the before and after situation. If you succeed then the symmetry does not exist. All the great conservation laws are associated with the assertion that a particular experiment is impossible. Indeed in the early 1900's Emmy Nöether showed that *every conservation law is associated with a certain invariance which in turn is associated with the statement of an impossible experiment.* For example, the conservation of angular momentum is associated with the statement that no experimentalist has been able to determine a preferred direction in space. A partial list of impossible experiments is given in Table 1.1.

Thus the existence of a symmetry tells us what is NOT possible but does not tell us what IS possible. The existence of a symmetry rules out some possibilities.[1] [2] It leads to *selection rules.* The existence of a symmetry constrains the form of theories used to model the system possessing an observed symmetry. We must strongly emphasise that *the existence of a symmetry can only be determined by experiment and is always a tentative statement.* We can never be sure that some improvement in experimental technique or some experiment not hitherto contemplated will reveal an inexactitude in the symmetry. As examples consider the parity violation experiment or the CP violation experiments of kaons.

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[1] Kepler in his beautiful Christmas essay *The Snowflake* is fascinated throughout by the existence of symmetry and cosmologically writes of the harmony of the spheres. Copernicus, prior to Kepler, recognises the approximate nature of symmetries - writing of the sphericity of the earth *Although it is not immediately apparent that it is a perfect sphere, because the mountains project so far and the valleys are so deep, they produce very little variation in the complete roundness of the Earth*

[2] Muslim theology sees only God as perfect and thus carpet chanters, recognising their own imperfection, will deliberately make the occasional error, so that such a carpet will contain imperfections which is then consistent with their theology.

■ **Table 1.1** Impossible experiments and symmetries.

Immeasurable Quantity	*Implied Invariance*	*Conserved Quantity*	*Accuracy*
Absolute Position	Space Translation	Momentum	exact(?)
Absolute Time	Time Displacement	Energy	exact(?)
Absolute Direction	Rotational	Angular Momentum	exact(?)
Relative Phase of charged and neutral particles	Charge Gauge Transformations	Charge Q	exact(?)
Left and Right Indistinguishability	Space Inversion P	parity	violated in weak interactions
Direction of Time Flow Indistinguishability	Time Reversal T	-	violated
Particle-AntiParticle Distinction	Charge Conjugation C	Charge Parity	violated in weak interactions
Relative phase of baryons and other particles	Baryon Gauge Transformations	Baryon Number B	exact(?)
Relative phase of e^- & ν_e and other particles	Electron Number Gauge Transformations	Electron Number $\mathcal{L}_e$	exact(?)
Relative phase of μ^- & ν_μ and other particles	Muon Number Gauge Transformations	Muon Number $\mathcal{L}_e$	exact(?)

■ 3. Broken symmetry

In practice very few symmetries are 'exact' and in most cases we are led to consider 'approximate' symmetries. A symmetry need not be exact to be useful. Indeed I would assert the following:

Proposition: *We should always strive to construct theories with the highest possible symmetry even if these are not exact symmetries of nature. The physics comes in the process of breaking the symmetry.*

■ 4. Global and local symmetries

A symmetry may be *global* or *local*. A local symmetry need not be global. In most of this course we will be discussing local symmetries.

■ 5. Types of symmetries

There are a wide range of possible symmetries that we might consider. Two major categories would be *discrete* and *continuous* symmetries. Discrete symmetries, such as reflections, inversions, time reversal, charge conjugation, parity, finite rotations, permutations etc. are associated with *multiplicative* or *phase-like* quantum numbers. Continuous symmetries such as translations and rotations are associated with *additive* quantum numbers (e.g. angular momentum $\mathbf{J}$ or linear momentum $\mathbf{p}$).

■ 6. Symmetry and the Universe

On a clear night, away from city lights, look up to the sky (A feat more readily accomplished in the time of Copernicus in old Toruń than in modern Toruń) and you arrive at two utterly amazing, and deep conclusions, concerning the nature of the Universe which are in accord with more detailed observations:-

1. *The universe is almost empty.*
2. *The universe is not empty.*

Matter in the universe is astonishingly rare. Radiation is in comparison superabundant there being about 10^{18} photons for every baryon[3]. Why is matter so rare? Why is there any matter in the universe? Or somewhat more anthropologically, Why can we ask these questions? Our ability to ask these questions hinges on their answer. Why is the matter in the universe predominantly of one type and does not appear in equal quantities of matter and antimatter? What is the origin of this broken symmetry between matter and antimatter? We shall return to these questions later.

■ 7. The charge neutrality of matter

How neutral is matter? What would happen if we placed all the protons of a 68kg person in one box and a meter away put all the electrons? We could anticipate that an attractive electric force would develop between the two groups of charges. Application of the Coulomb force law readily leads to the magnitude of the force as $\sim 10^{30}$ Newtons! Matter is neutral to an astonishing extent. This neutrality of matter hides from us the strength of the Coulomb force. From our daily experience with forces we falsely conclude that gravity is the strongest force when in fact it is the weakest of all known forces[4]. The neutrality of matter hints at another conservation law - namely charge conservation. What is the origin of charge conservation and the neutrality of matter[5]?

■ 8. The Beginnings of Particle Physics

The discovery of the particles of physics very much belongs to our century and can confidently be expected to continue into the next century. The only particle known up to the end of the last century was the *electron*, discovered in 1897 when J. J. Thomson measured the ratio of its charge to mass giving the first characterisation of an "elementary" particle. The next particle was the *photon* introduced by Einstein

[3] Questions of the existence of dark matter are irrelevant. Even if, as some believe, that 95% of the matter in the universe is unseen the existence of matter in the universe remains very rare.

[4] The separation of charges plays a key role in many human activities. A golfer propelling a golf ball by hitting it with a club involves *electrical forces* - gravity enters only in the subsequent motion of the struck ball

[5] In the 1950's R. A. Lyttleton suggested that the expansion of the universe could be explained if there was a slight difference in the charge on the proton from that on the electron. Subsequent precise measurements of the charge ratio have ruled out that possibility.

as a quantized packet of electromagnetic energy and as the basis for the quantum theory of light. The *proton* was characterised in 1907 with Thomson's measurement of the ratio of its charge to mass and finding it to be 1850 times the mass of the electron.

The concept of *spin* was introduced in 1925 by Uhlenbeck and Goudsmit and led to the electron and proton as spin $s = \frac{1}{2}$ particles. The introduction of spin gave a further means of classifying particles. Thus by 1925 the known particles were

Particle	Mass	Charge	Spin
Photon γ	0	0	1
Electron e	$0.510 MeV$	$-e$	$\frac{1}{2}$
Proton p	$938.2 MeV$	$+e$	$\frac{1}{2}$

■ 9. The Neutron and the Structure of Nuclei

No new particles were found until the 1932 discovery by Chadwick of the neutron. Iréne Curie and her husband F. Joliot had discovered that bombarding beryllium (Be) produced a very penetrating radiation which they suggested were very high energy $\gamma-$rays. Furthermore, they observed that the radiation interacted strongly with paraffin wax to produce protons. In the June 1 1932 issue of Proc.Roy.Soc. (London) Chadwick published his paper *Existence of a Neutron.* In the same issue some of Chawick's colleagues reported additional experimental evidence. In the June 20 1932 issue of the French journal Comptes Rendus Iréne Curie, F. Joliot and P. Savel reported new results, and following Chadwick, used the word neutron. Chadwick gained the Nobel Prize for his discovery - Curie and Joliot narrowly missed the discovery though later they won the Nobel Prize for their work on the production of artificial radioactivity. Heisenberg in the July 19 1932 issue of Zeitschrift fur Physik published Part I of his *Structure of Atomic Nuclei* attributing spin $s = \frac{1}{2}$ to the neutron. He claims that nuclei can be constructed from protons and neutrons only. Furthermore he says of the neutron "... if it consists of a proton and an electron, the electron must have zero spin and follow the Bose statistics. This is considered to be improbable, and the author would prefer to look upon the neutron as a fundamental unit.

Later experiments by Franco Rasetti unequivocally established the neutron as being a spin $\frac{1}{2}$ particle and that it must be regarded as distinct from the proton and electron. Thus in 1932 it appeared that all matter could be viewed as constructed from a nucleus of protons and neutrons with the electrons being in quantized orbits about the nucleus. Later physicists were to study the behaviour of protons and neutrons in magnetic fields and to measure their magnetic moments. Thus a particle could be characterised by the quantities

Mass	Charge	Spin	Magnetic Moment

The magnetic moment of the electron was measured in units of the Bohr magneton

$$\mu_B = \frac{e\hbar}{2m_e}$$

and nuclear magnetic moments in terms of the nuclear magneton

$$\mu_N = \frac{e\hbar}{2m_p}$$

the latter quantity being roughly $\frac{1}{1850}$ smaller than μ_B. Experiments were ultimately to determine these quantities with incredible accuracy as shown my the modern figures given below:

Particle	Mass	Charge	Spin	Magnetic Moment
Photon γ	$< 3 \times 10^{-27} eV$	$< 2 \times 10^{-32} e$	1	0
Electron e	$0.51099906 MeV$	$-e$	1/2	$1.001159652193 \mu_B$
Proton p	$938.27231 MeV$	$+e$	1/2	$2.79284739 \mu_N$
Neutron n	$939.56563 MeV$	$< 0.4 \times 10^{-21} e$	1/2	$-1.9130428 \mu_N$

Note that

$$\frac{\mu_p}{\mu_N} = -1.46 \approx -\frac{3}{2}$$

We normally think of a particle having to have a nett electric charge to have a magnetic moment and yet the apparently neutral neutron possess a magnetic moment. Furthermore why is the ratio of the proton to neutron magnetic moment approximately $-\frac{3}{2}$? These are two observations that a suitable theory of particles should explain. Likewise we should be able to explain why the mass of the neutron is just slightly greater than that of a proton.

With much greater difficulty we might expect to explain why the mass of the proton is approximately 1850 times that of the electron. I shall attempt to explain some of these observations in succeeding chapters. While it seemed apparent that all nuclei could be constructed from neutrons and protons and the existence of different isotopes of a given element understood there remained many fundamental questions concerning nuclei. Among these questions were:

1. What causes the protons and neutrons to be strongly bound in a nucleus?
2. Why do some nuclei undergo $\beta-$decay?
3. Why do some nuclei undergo $\alpha-$decay?
4. Why are some nuclei very stable while others are unstable?
5. Why does nuclear instability increase with increasing atomic number?
6. What is the mechanism that leads to $\beta-$decay?
7. What is the mechanism that leads to $\alpha-$decay?
8. Do protons and neutrons have a substructure or are they truly fundamental particles?
9. Are there any other fundamental particles? All nine of these questions were to be answered in the years following Chadwick's 1932 discovery of the neutron.

■ 10. Weyl, Dirac and the Positron

In 1928 Dirac introduced his relativistic equation for the electron and in 1932 C. D. Anderson experimentally discovered the anti-particle partner of the electron - the positron which had the opposite sign electric charge and magnetic moment. In 1929 Hermann Weyl tried to construct a relativistic two-component equation for describing a massless spin 1/2 particle. At that time no such particles were known. These two solutions separately described a left-handed and a right-handed particle. Pauli recognised that Weyl's equation violated space inversion symmetry, or parity, and dismissed Weyl's equation with the comment "God does not have a weak left-hand". Later (1957) with the downfall of parity Weyl's equation was revived. With the successful discovery of the positron Dirac's equation gained respectability and gave rise to the belief that there were two states of matter - matter and antimatter. This implied that for every particle there was an antiparticle and of the same mass but opposite charge and magnetic moment. Thus it was predicted that antiprotons and antineutrons should exist. These were not discovered until the 1950's when high energy accelerators were constructed.

■ 11. Handedness and Two ways of dividing an Apple

As an illustration of handedness consider the cutting of an apple into two equal halves. Our normal solution is to cut the apple with a single vertical cut to produce two symmetrical halves. There is another way to create two equal halves. First make a vertical cut to half way down, that is to the equator of the apple. Now turn the apple upside down and rotate it through 90^o and make a second vertical cut to the equator. Now make a horizontal cut to the centre of the apple along the equator starting at the point on the equator where one of the vertical cuts and cutting along the equatorial line for 90^o or, equivalently a quarter turn. You have a choice as to the direction you rotate your knife! Now rotate the apple to the position on the equator of the other vertical cut and make a second cut along the equator for 90^o moving in the same direction as the first horizontal cut. As long as you have been careful cutting to the centre of the apple your apple should separate into two equal halves. Now take a second apple and repeat the process but this time for the two horizontal equatorial cuts make the opposite choice you made when cutting the first apple. Now we have a further two halves of an apple. Can you fit a half from the first apple to a half of the second apple? You certainly could have if you had made the traditional cutting of the apples. What is the difference?

Chapter Eleven

More Particles

You boil it in sawdust: you salt it in glue;
You condense it with locusts and tape
Still keeping one principal object in view
To preserve its symmetrical shape.
— Lewis Carroll *The Hunting of the Snark*

1. SYNOPSIS

Progress in discovery new particles after 1932 proceeded very slowly. Most new results came from cosmic ray studies. It was not until 1956 that the design of accelerators started to permit the discovery of new particles. By 1960 major progress was being made on both the experimental and theoretical fronts. The possibility of a very light particle, the neutrino, was considered in 1934 and finally discovered in 1953. It had, and still is having, a major impact on our understanding of the universe.

■ 2. Charge conservation

Electrons do not appear to disappear. The experimental limit for the decay

$$e \not\rightarrow \nu + \gamma$$

is $> 1.5 \times 10^{25} yr$. Within these limits we know of no exception to the statement that *charge is conserved in all reactions* and hence we may label particles by their electric charge Q. We note that the difference in the absolute charge of the electron and proton is $< 10^{-21} e$.

■ 3. The $\beta-$ crisis

The discovery of $\beta-$decay gave rise to a serious problem in physics that appeared to threaten the very foundations of the subject. It was realised that for electron emission one had the reaction

$$\boxed{\boxed{{}^A_Z X \rightarrow {}^A_{Z+1} Y + e^-}} \tag{1}$$

The above equation conserved charge but appeared to violate conservation of energy and conservation of spin statistics. Whereas it was expected that the electron would be emitted at a single energy, corresponding to a transition between two quantized nuclear states, it was found that the energy distribution of the emitted electrons was continuous with low and high energy cutoffs. It appeared that energy was not being conserved.

The $\beta-$decay could be regarded as due to the transformation of a neutron into a proton and an electron

$$\boxed{\boxed{{}^1_0 n \rightarrow {}^1_1 p^+ + e^-}} \tag{2}$$

The proton, neutron and electron were all known to to be spin $s = \frac{1}{2}$ particles. The left-hand-side of Eq.(2) clearly can only have a total spin angular momentum of $S = \frac{1}{2}$ while the addition of the two spin $s = \frac{1}{2}$ particles on the right-hand-side of Eq. (2) can only yield total spin values of $S = 0,\ 1$. Thus it appeared that the particle on the left-side followed Fermi-Dirac statistics while the pair on the right-side would follow Bose-Einstein statistics. Physicists were very reluctant to give up the conservation of energy and of spin statistics and thus arose a major crisis.

■ 4. Pauli's Solution

A possible solution to the crisis was given by W. Pauli in a note he wrote declining an invitation to a Ball! He made no publication of his idea in the scientific literature! Pauli suggested that the crisis would be overcome if there existed a particle of very low mass with spin $s = \frac{1}{2}$ and zero charge. This at once saved the conservation of energy and preserved the spin statistics. The continuous nature of the electron emission could be understood as the energy available for the ejection of the electron would in fact be shared between the electron and the hypothesized particle. It was assumed that the particle had very little interaction with matter making it especially difficult to detect.

■ 5. Fermi's β−decay Theory

Enrico Fermi considered Pauli's suggestion and termed the hypothetical particle as the *neutrino*, the particle being very little and electrically neutral. In a landmark paper Fermi (*Zeitschrift für Physik* **88**, 161-71 (1934)) developed a theory of β−decay based upon Pauli's suggestion. He was able to explain the shape of the observed electron emission spectrum and suggested that a careful measurement of the low energy part of the spectrum could lead to constraints on the mass of the neutrino. The possibility of experimentally being able to detect the neutrino directly appeared in 1934 to be quite impossible. No sources generating large numbers of neutrinos were known. The intensity of neutrinos emitted from all known radioactive sources was much too low to hope for success. Direct detection would remain impossible unless a radically new source of neutrinos was found.

■ 6. Neutrinos and anti-Neutrinos

It followed from Dirac's relativistic wave equation for spin $\frac{1}{2}$ particles that if neutrinos exist so should antineutrinos. Symbolically these were designated as ν and $\bar{\nu}$ respectively and it was taken that *beta*−decay involved the emission of antineutrinos and hence Eq. (2) became written as

$$\boxed{{}^1_0n \rightarrow {}^1_1 p^+ + e^- + \bar{\nu}_e} \tag{3}$$

This raised the question *Are the neutrino and antineutrinos distinct particles?* The answer to this question would come much later.

■ 7. The Hunting of the Neutrino

It was the development of nuclear reactors that gave an entirely new and intense source of neutrinos, actually antineutrinos, the high flux coming from neutron decay as in Eq. (3) and additional neutrinos from β−decay of fission products. While the reactor is heavily shielded to contain the radioactivity in the reactor the antineutrinos are almost totally unaffected and penetrate the shielding with ease. Thus close to the outside of a high flux reactor is an ideal site for attempting to detect antineutrinos. The basic idea was to have the antineutrino induce β−decay via the reaction

$$\boxed{\bar{\nu} + p \rightarrow n + e^+} \tag{4}$$

Such a process could be identified by observing the emitted positron followed by the γ−rays emitted by the annihilation of the positron by an electron

$$\boxed{e^+ + e^- \rightarrow 2\gamma} \tag{5}$$

In that way Reines and Cowan in 1953 were able to announce the detection of antineutrinos (F. Reines and C. L. Cowan, Jr. *Phys. Rev.* **90**, 492 (1953)). This was to be the start of a whole series of experiments elucidating the properties of neutrinos and which continue to this day.

■ 8. Symmetry Again

Experiments involving neutrinos forced physicists to sharpen their ideas on symmetry and in particular the symmetries associated with Parity $\mathcal{P}$, Charge conjugation $\mathcal{C}$ and Time reversal $\mathcal{T}$. Each of these symmetries involve a *discrete* transformation and we now will consider them separately.

■ 9. Parity $\mathcal{P}$

In chapter nine we mentioned briefly inversion symmetry in regard to a square in two-dimensions. There we saw it was equivalent to a point symmetry rotation through 180^o. In three-dimensions the situation is quite different. A spatial inversion cannot be reduced to a set of rotations in 3-space. By a spatial inversion $\mathcal{I}$ we mean a symmetry transformation $\mathcal{P}$ such that

$$(\mathbf{r}, t) \xrightarrow{\mathcal{P}} (-\mathbf{r}, t) \tag{6}$$

The operator P is commonly referred to as the *Parity* operator NB. The parity operator is very different from that of the angular momentum operator. The former is associated with multiplicative eigenvalues while the latter is associated with additive eigenvalues. The parity operator is always a discrete operator whereas the angular momentum can be a continuous operator. Angular momentum conservation arises from the assumption that there is no preferred direction in space. Spatial inversion is a less obvious property of space and indeed less fundamental.

Under $\mathcal{P}$

$$\mathbf{r} \xrightarrow{\mathcal{P}} -\mathbf{r} \quad \text{and} \quad \mathbf{p} \xrightarrow{\mathcal{P}} -\mathbf{p} \tag{7}$$

The angular momentum operator $\ell = \mathbf{r}\ \mathbf{x}\ \mathbf{p}$ is of *even* parity since under $\mathcal{P}$

$$\mathbf{r} \xrightarrow{\mathcal{P}} -\mathbf{r} \quad \text{and} \quad \mathbf{p} \xrightarrow{\mathcal{P}} -\mathbf{p} \tag{8}$$

Likewise, spin and charge are even parity operators whereas the electric field $\mathcal{E}$ is of *odd* parity. Recalling that

$$\mathcal{B} = \nabla \times \mathbf{A} \quad \text{and} \quad \mathcal{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t} \tag{9}$$

we may conclude that the magnetic field vector $\mathcal{B}$ is of *even* parity.

■ 10. Left-handed and Right-handed neutrinos

We may use the parity symmetry to describe massless neutrinos. Under $\mathcal{P}$ the direction of the linear momentum $\mathbf{p}$ is reversed but the spin is unchanged. If the neutrino is massless then we expect its spin to point either parallel or antiparallel with respect to its momentum direction and the momentum direction to be the direction of propagation of the neutrino. If the spin and momentum are parallel we shall term the particle *Right-handed* and if antiparallel *Left-handed* Thus we expect neutrinos ν_L and ν_R and antineutrinos $\bar{\nu}_L$ and $\bar{\nu}_R$ - if parity is conserved!

■ 11. Time-reversal $\mathcal{T}$

Under time reversal $\mathcal{T}$

$$(\mathbf{r}, t) \xrightarrow{\mathcal{T}} (\mathbf{r}, -t) \tag{10}$$

and hence the motion of objects is reversed. As a consequence the time reversal operation changes the sign of angular momentum operators and since the directions of currents are reversed so is the direction of the magnetic field $\mathbf{B}$.

■ 12. Charge conjugation $\mathcal{C}$

The operation of *Charge conjugation* C replaces a given particle by its antiparticle. Thus $\mathcal{C}$ changes the sign of all charges $(\mathcal{Q}, \mathcal{B}, \mathcal{L}, \mathcal{S})$. The electric field $\mathcal{E}$ is associated with static charges while the magnetic field $\mathcal{B}$ is associated with currents and hence

$$\mathcal{E} \xrightarrow{\mathcal{C}} -\mathcal{E} \quad \text{and} \quad \mathcal{B} \xrightarrow{\mathcal{C}} \mathcal{B} \tag{11}$$

We summarise the operations of $\mathcal{C}, \mathcal{P}, \mathcal{T}$ in Table 1.

Table 1. Transformations under $\mathcal{C}, \mathcal{P}$ and $\mathcal{T}$.

Quantity	$\mathcal{C}$	$\mathcal{P}$	$\mathcal{T}$
$\mathbf{r}$	$\mathbf{r}$	$-\mathbf{r}$	$\mathbf{r}$
t	t	t	$-t$
$\mathbf{p}$	$\mathbf{p}$	$-\mathbf{p}$	$-\mathbf{p}$
$\mathbf{L} = \mathbf{r}\ \mathbf{x}\ \mathbf{p}$	$\mathbf{L}$	$\mathbf{L}$	$-\mathbf{L}$
$\mathbf{S}$	$\mathbf{S}$	$\mathbf{S}$	$-\mathbf{S}$
$\mathcal{E}$	$-\mathcal{E}$	$-\mathcal{E}$	$\mathcal{E}$
$\mathcal{B}$	$-\mathcal{B}$	$\mathcal{B}$	$-\mathcal{B}$
$\mathcal{Q}$	$-\mathcal{Q}$	$\mathcal{Q}$	$\mathcal{Q}$

■ 13. The $\mathcal{CPT}$ theorem

Schwinger, Lüders and Pauli have established a remarkable theorem known as the $\mathcal{CPT}$ theorem which states that any quantum field theory *compatible with special relativity* and which *assumes only local interactions* is invariant under the combined action of $\mathcal{CPT}$ including all orderings of the three operators. This means that while there may be noninvariance with with any of the individual operators there cannot be, within the assumptions of the theorem, noninvariance with the triple product. Thus if there is noninvariance with respect to $\mathcal{CP}$ then there will consequently be noninvariance with respect to $\mathcal{T}$ but not for $\mathcal{CPT}$.

As an example consider the action of $\mathcal{C}$ and $\mathcal{P}$ on the left-handed neutrino ν_L as illustrated below. Under the action of $\mathcal{P}$ the left-handed neutrino is turned into a right-handed neutrino ν_R while under charge conjugation it is turned into left-handed antineutrino $\bar{\nu}_L$. However, under the joint action of $\mathcal{CP}$ we obtain a right-handed antineutrino $\bar{\nu}_R$.

$$\begin{tikzcd}
\nu_L \arrow[r, "\mathcal{P}"] \arrow[d, "\mathcal{C}"'] & \nu_R \arrow[d, "\mathcal{C}"'] \\
\bar{\nu}_L \arrow[r, "\mathcal{P}"] & \bar{\nu}_R
\end{tikzcd}$$

We have obtained a beautifully symmetric figure showing all the actions of the three transformations $\mathcal{C}, \mathcal{P}, \mathcal{T}$ and seemingly a complete picture - but remember H. L. Mencken

For every complex question there is a simple answer
— and it's wrong.
— H. L. Mencken

Where have we gone wrong! We have assumed that Nature has produced all the particles in our diagram BUT there is no right-handed neutrino and no left-handed antineutrino!

Under the action of $\mathcal{P}$ the left-handed neutrino is turned into the non-existent right-handed neutrino ν_R while under charge conjugation it is turned into the non-existent left-handed antineutrino $\bar{\nu}_L$. However, under the joint action of $\mathcal{CP}$ we obtain the observed right-handed antineutrino $\bar{\nu}_R$. Since $\mathcal{CP}$ invariance is maintained in weak interactions then if the $\mathcal{CPT}$ theorem holds then weak interactions would also be time reversal $\mathcal{T}$ invariant.

Thus Nature has chosen the parity symmetry to be broken but $\mathcal{CP}$ and $\mathcal{T}$ to be conserved in the weak interactions associated with $\beta-$ decay. As we shall see later, in kaon physics even $\mathcal{CP}$ symmetry is broken and if the $\mathcal{CPT}$ theorem holds that would imply that time-reversal symmetry is also broken. Thus consideration of the properties of the neutrinos can lead to profound consequences for the physicists conception of the universe.

Chapter Twelve

Leptons

It is very rare that any major new insight into the natural world has come about inductively by the Baconian method of assembling large sets of data and deriving general laws from them. Far more typically a flash of imagination based on very few observations, leads to a theoretical structure, usually in the form of a mathematical system by which the results of further observations can be calculated. Those calculations are then compared with observation and if there is satisfactory agreement, the theoretical model is accepted as a means of predicting yet other observations

A. Cook, *The Observational Foundations of Physics* Cambridge University Press (1994)

1. SYNOPSIS

The discovery of the electron and its associated neutrino was followed by the discovery of other objects with similar properties, apart from mass. Such objects form families termed *leptons.* New quantum numbers, known as *lepton numbers*, are introduced. Three families of leptons are known and it is believed the list of possible lepton families is complete. The electro-weak theory of Glashow, Salam and Weinberg unified electromagnetism with the weak interactions associated with $\beta-$decay led to the prediction that weak interactions involve a triplet of bosons $W^{\pm}$, Z^0.

■ 2. Yukawa's Nuclear Force Hypothesis

The discovery of the electron, proton and neutron lead to the picture of the atom as a nucleus containing Z protons and N neutrons surrounded by electrons in quantized orbitals (Z electrons in the case of neutral atoms). The nucleus had a radius $\sim 10^{-15}m$ compared with an atomic radius $\sim 10^{-10}m$. With the introduction of Dirac's relativistic wave function it became possible to make surprisingly precise predictions of the electronic properties of many atoms. For the electrons the predominant interactions involved the Coulomb attraction between electrons and the positively charged nucleus tending to pull the electrons into the nucleus but compensated by the inter-electron repulsion.

No such model of the nucleus existed in 1935. It was clear that in spite of the Coulomb repulsion between pairs of protons the neutrons and protons were bound to one another by a strong non-Coulombic nuclear force. This force appeared to be essentially independent of the charge of the nucleons.

The first theory of a strong nuclear force came from Japan with H. Yukawa's paper *Interaction of Elementary Particles. Part I* which was published in English in Proc. Phys. Math. Soc. Japan **17** , 48-57 (1935). Yukawa assumed that the strong nuclear force arose from the exchange of hitherto unknown particles, later to be called *mesons.* The strong force arose from pairs of nucleons tossing back and forth mesons. These were assumed to be bosons, to preserve spin statistics, and to have three possible charge states $+e,\ 0, -e$. The resultant force was assumed to be, unlike the Coulomb force, a *short range* force acting only over nuclear dimensions. This meant that the mesons must be confined to within the nucleus. Using Heisenberg's uncertainty principle it was possible to deduce that the mesons would have a mass of $\sim 300m_e$. Thus Yukawa's meson theory involved a triplet of particles which we shall designate as

$$\pi^- \qquad \pi^0 \qquad \pi^+ \tag{1}$$

No such particles were known in 1935.

Yukawa showed that his meson forces could be described by a potential

$$Y(r) = -g\frac{e^{-kr}}{r} \tag{2}$$

where g and k were appropriate constants. The Yukawa potential was very different from the Coulomb potential associated with the electrons of atoms. The constants g, k were fixed so that at distances greater than the nuclear radius the potential went to zero whereas at distances less than the nuclear radius the potential fell rapidly forming a deep potential well leading to confinement of the nucleons.

■ 3. Discovery of the Muons

In 1935 no accelerators were available to give any possibility of creating Yukawa's mesons. The only source of high energy particles came from the study of cosmic rays. In 1936 C. D. Anderson and S. Neddermeyer observed some tracks left in a photographic emulsion a collision of a cosmic ray particle. The new particle was termed a mesotron as it was thought to be one of Yukawa's mesons. Subsequent measurements showed it was a particle of spin $\frac{1}{2}$ and hence could not be a Yukawa meson. The particle was finally named the *muon* and designated as μ^-. Apart from it having a mass of about $210m_e$ and a relatively short lifetime it seemed to have properties almost identical to those of an electron. I. Rabi, on learning this, remarked "Who ordered this one!" Like the electron it also had its corresponding anti-particle, the anti-muon, $(\bar{\mu}^+)$ the analogue of the positron.

Particle	Q	mass	spin	mag. mom.	mean life
electron	$-e$	$0.511MeV$	$\frac{1}{2}$	$1.00116\mu_B$	$> 2.7 \times 10^{27}yr$
muon	$-e$	$105.7MeV$	$\frac{1}{2}$	$1.00116\frac{e\hbar}{2m_\mu}$	$2.2 \times 10^{-6}s$

We would expect the muon to be able to decay via the following processes

$$\mu^- \rightarrow e^- + \gamma \tag{3a}$$
$$\mu^- \rightarrow e^- + e^- + e^+ \tag{3b}$$
$$\mu^- \rightarrow e^- + 2\gamma \tag{3c}$$

Such decays can certainly satisfy spin, charge, energy and momentum conservation laws but there is no experimental evidence for such decays. What is preventing such decays from occurring?

The observed decay of the muon is of the form

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu \tag{4a}$$

and the anti-muon as

$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu \tag{4b}$$

In each case two neutrinos are emitted and they differ from one another! The electron and muon neutrinos are experimentally distinguishable!

■ 4. Leptons

The observation of the electron and muon together with their respective neutrinos suggests the existence of two families of light fermions (spin $\frac{1}{2}$)

$$\begin{pmatrix} e^- \\ \bar{\nu}_e \end{pmatrix}, \qquad \begin{pmatrix} e^+ \\ \nu_e \end{pmatrix} \tag{5a}$$

and

$$\begin{pmatrix} \mu^+ \\ \bar{\nu}_\mu \end{pmatrix}, \qquad \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix} \tag{5b}$$

These two families of particles are known as *leptons*. The leptons were originally thought of as a set of particles all of mass less than that of the proton.

■ 5. The Four Forces

The protons and neutrons were seen as strongly interacting particles whereas the leptons appeared to be weakly interacting particles. This led to the idea that there existed in nature four distinct types of forces and that particles could be classified by their response to these forces. The four forces are referred to as:-

1. The *gravitational force* which is the weakest of the four and has an infinite range of action. It is a purely attractive force.
2. The *weak force* which is of very short range and is responsible for $\beta-$decays.

3. The *electromagnetic force* which is vastly stronger than the gravitational or weak forces. It can be either attractive or repulsive according to the signs of the charges involved. Like the gravitational force it appears to have an infinite range of action.
4. The *strong force* which is also a short range force and is associated with strongly interacting particles such as protons and neutrons.

All known particles appear to respond to the gravitational force whereas leptons respond to the weak and electromagnetic forces but evidently not to the strong force.

■ 6. Lepton Numbers and Lepton Conservation

The failure of many decays involving leptons to occur even though the decays seem to satisfy all known conservation laws makes one suspect that there is associated a hitherto unknown conserved quantum that we shall call the *lepton number* L which is zero for all particles that are *not* leptons while for a lepton it has the value ± 1. Let us choose $L = 1$ for the electron. For a neutron, proton or photon (γ) $L = 0$. Consider the decay of a neutron in free space

$$n \rightarrow p + e^- + \bar{\nu}_e \tag{6}$$

The left-hand-side of Eq (6) has lepton number $L = 0$ and hence the lepton numbers on the right-hand-side must sum to 0 if there is lepton conservation. This will be assured if we assign $L = -1$ to the electron anti-neutrino $\bar{\nu}_e$.

Likewise the electron-positron annihilation

$$e^- + e^+ \rightarrow 2\gamma \tag{7}$$

requires that the positron has $L = -1$ and we must also take the neutrino, ν_e, with $L = +1$.

We might then conclude that similar assignments of lepton numbers to the muon and its neutrino would hold. But the muons do not decay as in Eq (3a-c), but rather as in Eq (4a,b) ie.

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu \tag{4a}$$

and the anti-muon as

$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu \tag{4b}$$

This implies that the muon has its own lepton numbers and are distinct from those of the electron. Thus we are forced to associate lepton numbers L_e with the four members of the electron lepton family, Eq (5a), and different lepton numbers L_μ with the muon lepton family, Eq (5b). Attaching lepton numbers to the lepton families we have the two families

$$\begin{pmatrix} e^-, & L_e = +1 \\ \bar{\nu}_e, & L_e = -1 \end{pmatrix} \qquad \begin{pmatrix} e^+, & L_e = -1 \\ \nu_e, & L_e = +1 \end{pmatrix} \tag{8a}$$

and

$$\begin{pmatrix} \mu^-, & L_\mu = +1 \\ \bar{\nu}_\mu, & L_\mu = -1 \end{pmatrix} \qquad \begin{pmatrix} \mu^+, & L_\mu = -1 \\ \nu_\mu & L_\mu = +1 \end{pmatrix} \tag{8b}$$

■ 7. More Lepton Families?

By the end of the 1960's the two lepton families were well established and in the early 1970's the question "Are there any more lepton families?" was being asked. With stunning successes coming from the quark theory of the strongly interacting particles predictions of the possible existence of a third family of leptons were being considered. The completion of the two mile Stanford Linear Accelerator was making possible the study of electron- positron annihilation at hitherto unobtainable energies. The annihilation process resulted in the creation of a packet of electromagnetic energy that could decay into photons or particle-antiparticle pairs

$$e^+ + e^- \rightarrow ??$$

1976 saw the production of a pair of heavy leptons each of mass $1777.1 MeV$ being almost twice the mass of the proton. This lepton became known as the *tauon* (τ). It presumably had associated with it a neutrino ν_τ. It is clear that this is a separate family of leptons with its own conserved lepton numbers. The lifetime of the τ was found to be very short with a mean life of $\sim 3 \times 10^{-13} s$.

Thus the third family of leptons

$$\begin{pmatrix} \tau^-, & L_\tau = +1 \\ \bar{\nu}_\tau, & L_\tau = -1 \end{pmatrix} \qquad \begin{pmatrix} \tau^+, & L_\tau = -1 \\ \nu_\tau & L_\tau = +1 \end{pmatrix} \tag{8c}$$

was established.

■ 8. No More Families of Leptons?

Having found three families of leptons is the set of lepton families complete? Are more families likely to be found? There are good theoretical and experimental grounds for thinking that there are no more families of leptons. Cosmological considerations of the amount of helium and lithium in the universe are compatible with fewer that four families. A critical experiment at CERN, which I will refer to in a later chapter, gives the number of families as

$$N = 2.983 \pm 0.025$$

so perhaps the subject of the number of families is closed as is the subject of the number of magic hexagons.

■ 9. The Magic Hexagon – A Course Irrelevancy

		15		
	14		13	
9		8		10
	6		4	
11		5		12
	1		2	
18		7		16
	17		19	
		3		

Every row of the numbers in the above hexagon adds up to the same total (38).

This pattern was discovered by the American railway clerk, Clifford Adams. Without knowing whether it was possible to make any magic hexagon, he began his search in 1910. He used a set of ceramic hexagon tiles marked with the numbers from 1 to 19. Again and again he tried arranging these in different ways so that all the rows added up to the same number. He worked at the problem on and off for 47 years before discovering this arrangement while recovering from an operation.

Unfortunately he then lost the piece of paper on which he had written the solution. He was unable either to remember it or reconstruct it. But five years later in December 1962 he found the missing piece of paper and sent his magic hexagon to the mathematics writer Martin Gardner. Gardner passed it on to Charles Trigg who revealed that no one had ever discovered a magic hexagon before. Furthermore, Trigg was able to prove that no other magic hexagon of any size was possible.

Source:- Richard Phillips, *Numbers, facts, figures and fiction* Cambridge University Press, (1994).

■ 10. Some Questions

1. Can you construct an analogue of the magic hexagon in three dimensions? (I don't know).
2. Take 24 identical coins and arrange them in four rows of 6 so that there are 6 columns of 4 coins. Draw a rectangle around the set of coins. Take a further coin and show that you can rearrange the coins so that 25 coins fit inside the rectangle. (25 20groszy coins are ideal).

The will to learn is an intrinsic motive, one that finds both its source and its reward in its own exercise. The will to learn becomes a 'problem' only under specialized circumstances like those of a school, where a curriculum is set, students confined, and a path fixed. The problem exists not so much in learning itself, but in the fact that what the school imposes often fails to enlist the natural energies that sustain spontaneous learning - curiosity, a desire for competence, aspiration to emulate a model, and a deep-sensed commitment to the web of social reciprocity

J. S. Bruner, *Toward a Theory of Instruction* Harvard University Press (1966)

Chapter Thirteen

The Mesons

On a cosmic scale, we are still awakening to the larger world around us. Yet the power of even our limited intellect is such that we can abstract the deepest secrets of nature.

Does this give meaning or purpose to life?

Some people seek meaning in life through personal gain, through personal relationships, or through personal experiences. However, it seems to me that being blessed with the intellect to divine the ultimate secrets of nature gives meaning enough to life.

M. Kaku

SYNOPSIS

Yukawa's meson was eventually discovered in cosmic ray studies. However, this was just the start of the discovery of many other mesons. What role do these play? Can we classify the mesons into any meaningful patterns? The octets and nonets. Why octets and nonets?

■ 1. Yukawa's Mesons Found

Yukawa's prediction of the existence of three mesons being involved in his theory of the strong nuclear binding force received support in 1947 with their observation by Lattes, Occhialini and Powell in photographs exposed at high altitudes to cosmic rays. The following year the Berkeley cyclotron reached sufficiently high energies to be able to produce Yukawa's mesons in a controlled manner so that their properties could be examined in detail. The three mesons were collectively called *pions*.

■ 2. Properties of the Pions

Yukawa had predicted a mass of about 300 times that of the electron. Detailed measurements yielded the masses as

$$\pi^{\pm} = 139.56995 MeV \qquad \pi^0 = 134.9764 MeV \tag{1}$$

The $\pi^{\pm}$ pions were expected to constitute a particle-antiparticle pair of particles and by the CPT theorem were expected to be of the same mass. There was no reason to expect equality with the mass of the neutral pion π^0 as indeed observed.

The pions were observed to be highly unstable particles with mean lives of

$$\pi^{\pm} \quad 2.6030 \times 10^{-8} s \qquad \pi^0 \quad 8.4 \times 10^{-17} s \tag{2}$$

Note again the important difference between the charged and neutral pions. The π^0 has a *very* much shorter mean life than the charged $\pi^{\pm}$ pions.

The charged pions decay almost entirely (to $\sim 99.9877\%$) via the reaction

$$\pi^+ \rightarrow \mu^+ + \nu_\mu \tag{3}$$

with the negative pion decaying into the charge conjugates of Eq(3). The neutral pion π^0 decays almost entirely (to $\sim 98.798\%$) as

$$\pi^0 \rightarrow 2\gamma \tag{4}$$

The fact that the π^0 decays into a *pair* of γs each of which is a spin 1 particle shows that the π^0 must have an integer spin and hence be a *boson* as predicted by Yukawa. Detailed consideration of the rotation and reflection properties of the γ pair rules out *odd* spin values leaving just the possibility of $S = 0$ or 2. The assignment of $S = 0$ for the π^0 is consistent with the experimental observation of a null magnetic moment as would be expected for a neutral spin zero particle.

In the case of the $\pi^{\pm}$ pions there is direct experimental evidence for $S = 0$ and we may conclude that the pions form a triplet of particles with $S = 0$ i.e. three spinless bosons.

The observation of the properties of the pions raises several important questions.

1. Why are the masses $m_{\pi^\pm} \sim m_{\pi^0}$ (to approximately 3%) ?
2. Why is the mean life of the π^0 so very much shorter than that of the $\pi^\pm$?
3. Are there any other mesons?
4. Could there be mesons with spin $S = 1$ *vector mesons* ?
5. What is the relationship of the mesons to other particles such as the proton and neutron?
6. Are the mesons truly elementary particles devoid of substructure or are they composites of some more fundamental particles ?

■ 3. Laboratory Production of Pions

Intense beams of pions can be created in the laboratory, usually by nucleon-nucleon collisions as typified by the reactions

$$
\begin{aligned}
p + p &\rightarrow p + p + \pi^0 \\
p + p &\rightarrow p + n + \pi^+ \\
p + n &\rightarrow p + p + \pi^- \\
p + n &\rightarrow p + n + \pi^0 \\
p + n &\rightarrow n + n + \pi^+
\end{aligned} \tag{5}
$$

Thus one has meson factories such as Triumf (in Vancouver) that are dedicated to producing controlled beams of pions.

Mesons may also be produced by allowing a beam of high energy γ's impinge on protons to produce

$$
\begin{aligned}
\gamma + p &\rightarrow p + \pi^0 \\
\gamma + p &\rightarrow n + \pi^+
\end{aligned} \tag{6}
$$

■ 4. Isospin and the Pions

We saw earlier that the mass of the proton was almost the same as that of the neutron and this led Heisenberg to suggest that the neutron and proton could be regarded as two charge states of a single particle - the nucleon. It was suggested that the nucleon could be considered to be a particle having an *isospin* $I = \frac{1}{2}$ by analogy with the spin of the electron. The proton was assigned an isospin projection of $I_z = +\frac{1}{2}$ and the neutron $I_z = -\frac{1}{2}$. Thus the proton and neutron were grouped together as an isospin *multiplet*, in this case an isospin *doublet*.

A similar situation arose with the pions. Here there were three particles of approximately the same mass so could they be treated as members of an isospin triplet with $I = 1$? In that case the π^+ is assigned an isospin projection of $I_z = +1$ with π^- having $I_z = -1$ as would be expected for a particle-antiparticle pair. Then the π^0 must have $I_z = 0$. Without some underlying principle these assignments appear arbitrary. Were isospin an *exactly* conserved quantity we would expect the masses of the particles belonging to a given isospin multiplet to have *exactly* the same mass. This is almost but not *exactly* the case. Could it be that in certain reactions isospin is approximately conserved?

■ 5. Isospin Conservation

The photon is associated with electromagnetic reactions and is assigned $I = 0$. The reaction

$$
\pi^0 \rightarrow 2\gamma \tag{7}
$$

clearly does NOT conserve isospin whereas the reactions in Eqs (5) and (6) DO conserve isospin. That reaction involves photons which is a characteristic of *electromagnetic* interactions.

The reaction

$$
\pi^+ \rightarrow \mu^+ + \nu_\mu \tag{8}
$$

is entirely analogous to the *weak* interactions that characterise $\beta-$decay. Weak interactions are much weaker than electromagnetic interactions and it is due to that fact that results in the mean life of the π^0 being very much *shorter* than for the decay of of the charged pions. The pions react *strongly* with protons and neutrons as in the reaction

$$\pi^- + p \rightarrow \pi^0 + n \tag{9}$$

The observations summarized by the above three equations are consistent with the statement

> *Isospin is conserved in all strong interactions but is broken by weak and electromagnetic interactions*

This is consistent with the assignment of isospin $I = 0$ to all particles that do not involve strong interactions, i.e. for all *leptons* and the *photon*.

■ 7. Enter the Hadrons

The strongly interacting particles form a set of particles distinct from the leptons and the photon. All particles that involve the strong force are termed *hadrons* and include both the pions AND the nucleon. Hadrons, as a class, include both *bosons* and *fermions*. It is believed that IF one could turn off the weak and electromagnetic interactions then the particles belonging to a given isospin multiplet would be degenerate, i.e. have exactly the same mass. Thus a mass splitting in an isospin multiplet is symptomatic of isospin symmetry breaking via the weak and/or electromagnetic interactions.

Our particles now include the photon, in a class by itself, the leptons and the hadrons with the latter including the pions and nucleons. The pions identify with Yukawa's mesons. Is the picture complete? We have introduced isospin as an additional quantum numbers - are there further quantum numbers to be exposed? Is there a quantum number that distinguishes different isospin multiplets.

■ 8. Baryons and Mesons

The hadron class of particles involves both fermions and bosons. The bosonic hadrons are termed *mesons* and the fermionic hadrons are termed *baryons*.

■ 9. Strange Events

In 1947 Rochester and Butler used a cloud-chamber to study cosmic rays and observed a particularly strange event. Tracks were observed in the form of a V. One track was identified as a proton and the other as a π^-. Charge conservation would imply that they came from the decay of an unknown neutral particle Λ^0

$$\Lambda^0 \rightarrow p + \pi^- \tag{10}$$

With the development of particle accelerators it became possible by 1954 to bombard pions on to a proton target. More strange events were observed. In particular

$$\pi^- + p \rightarrow \Lambda^0 + K^0 \tag{11}$$

with the two neutral particles subsequently decaying as

$$\Lambda^0 \rightarrow p + \pi^- \tag{12}$$

$$K^0 \rightarrow \pi^+ + \pi^- \tag{13}$$

The spin conservation leads to the conclusion that the Λ^0 particle has $S = \frac{1}{2}$ while the K^0 had $S = 0$. If they were hadrons then this meant a new baryon (Λ^0) and a new meson (K^0) had been discovered. However, things were very strange. The cross-sections measured for Eq(11) were consistent with a process occuring via strong interactions whereas the subsequent decays had mean lifes much longer than expected - indeed consistent with weak interactions. This suggested that there must be a hitherto unrecognized quantum number, appropriately termed *strangeness* $\mathcal{S}$ that was conserved in strong interactions but violated in weak and electromagnetic interactions. If Eq (11) is to conserve strangeness and the p and π are non-strange particles with $\mathcal{S} = 0$ then Λ^0 and K^0 must have *opposite* strangeness. since the strangeness on the right-hand-side of Eq.(11) must sum to zero. This will be the case if we arbitrarily assign $\mathcal{S} = -1$ to the Λ^0 baryon and $\mathcal{S} = +1$ to the K^0 meson. The reactions in Eq (12) then have

$$\Delta|\mathcal{S}| = 1 \tag{13}$$

and violate strangeness conservation as expected.

10 Further Baryons and Mesons

One of the great achievements of the 1950's was the controlled production of many new baryons and mesons using increasingly high energy accelerators and improved detection systems. In particular, the 1952 completion of the Brookhaven accelerator that could produce 2.3GeV protons followed by the Berkeley Bevatron that produced 6.2GeV protons, and Glaser's 1952 production of the liquid hydrogen bubble chamber. The Bevatron was able to create the antiproton $\bar{p}$ via the reaction

$$p + p \to p + p + p + \bar{p} \tag{14}$$

This was fully expected. Completely unexpected was the production of many new baryons and mesons.

11. Further Isospin Multiplets of Baryons and Mesons

The proton and neutron had been grouped as an isospin doublet and the pions as an isospin triplet. With the discovery of further baryons and mesons it was natural to attempt to also organise them into isospin multiplets by looking for particles of nearly the same mass. No charged partners were found for the Λ^0 baryon so it was classified as an isospin singlet ($I = 0$) with spin $S = \frac{1}{2}$. A set of three baryons, $(\Sigma^-, \Sigma^0, \Sigma^+)$, were found with masses

$$m_{\Sigma^-} = 1197.44MeV, \qquad m_{\Sigma^0} = 1192.04 \qquad m_{\Sigma^+} = 1189.37MeV \tag{15}$$

and strangeness $\mathcal{S} = -1$ and mean lives $\sim 10^{-20}s$. These three particles were thus assigned $I = 1$.

A pair of baryons (Ξ^-, Ξ^0) with masses

$$m_{\Xi^0} = 1314.9MeV \qquad m_{\Xi^+} = 1321.3MeV \tag{16}$$

were observed. The Ξ^- particle underwent a two stage decay

$$\begin{aligned} \Xi^- &\to \Lambda^0 + \pi^- \\ \Lambda^0 &\to p + \pi^- \end{aligned} \tag{17}$$

each step involved a change in $\mathcal{S}$ of one unit and hence the isospin doublet Ξ^-, Ξ^0 must have strangeness $\mathcal{S} = -2$.

The Λ^0 particle has a mass

$$m_{\Lambda^0} = 1115.5MeV \tag{18}$$

with isospin $I = 0$ and strangeness $\mathcal{S} = -1$. Thus the two isospin multiplets $\Sigma^-, \Sigma^0, \Sigma^+$ and Λ^0, with $I = 1$ and $I = 0$ respectively, are both of strangeness $\mathcal{S} = -1$ and have nearly the same mass. Can this be an accident? Is there an approximate symmetry higher than that of isospin that can bring together two different isospin multiplets, but of the same strangeness, into some supermultiplet? This will be the subject of the next chapter.

Exercises

1. Make a plot of the average mass of each baryon isospin multiplet along a vertical axis with the charge Q along the horizontal axis.
2. Write $Y = \mathcal{S} + 1$ and make a plot of Y along the vertical axis and isospin projection I_z along the horizontal axis marking on your graph the coordinates Y, I_z of each of the eight baryons together with their symbol.
3. If Q is the baryon electric charge in units of $+e$ can yuo find a simple relationship giving the charge of every particle in the graph of Ex. 2 in terms of its Y, I_z quantum numbers. If you do you will have discovered the Gell-Mann - Nishijima formula in the same way as did its discoverers.

Summary of Properties of low mass Spin $\frac{1}{2}$ Baryons

Baryon	Mass	$\mathcal{S}$	I	I_z	Q
p	938.3	0	$\frac{1}{2}$	$\frac{1}{2}$	1
n	939.6			$-\frac{1}{2}$	0
Σ^+	1189.4	- 1	1	+ 1	1
Σ^0	1192.6			0	0
Σ^-	1197.4			- 1	- 1
Λ^0	1115.7	-1	0	0	0
Ξ^0	1314.9	-2	$\frac{1}{2}$	$\frac{1}{2}$	0
Ξ^-	1321.3			$-\frac{1}{2}$	- 1

Predicting the Future

"I think there is a world market for maybe five computers," said Thomas J. Watson Snr, chairman of IBM, just before the first true electronic computer, ENIAC, came into use 50 years ago.

DEC's founder, Ken Olsen, proffered another of the industry's famous last words: "There is no reason anyone would want a computer in their home," he said.

Referring to the amount of memory PC software running under Dos would need, Microsoft's chairman Bill Gates said in 1981: "640k ought to be enough for anybody".

Breakthroughs by Charles Panati (Boston: Houghton Mifflin 1980)
Astonishing advances coming in your lifetime, in medicine, science and technology. By 1984 a liquid will spray away tooth decay; by 1989 physicists will have harnessed fusion power, a clean and near-limitless energy source; by 1994 hurricanes will be tamed and production of rainfall over arid lands will be commonplace. And more: electric cars, flying trains, drugs to prolong life, people/plant hybrids and awakening the (clinically) dead.
" A ten-course banquet for the imagination" NEW YORK TIMES

Chapter Fourteen

Patterns of Hadrons

No man is wise enough to think of all the ideas that can occur to a fool.

Rudolp Peierls *Birds of Passage* Princeton University Press (1985)

SYNOPSIS

One of the great achievements of the late 1950's and early 1960's was to group the various hadrons, baryons and mesons into meaningful patterns. Incomplete patterns indicated missing particles which gave motivation for their discovery, the high point being the discovery of the predicted Ω^- particle.

■ 1. The Baryon Number $\mathcal{B}$

The proton is an incredibly stable particle with a lifetime $>\sim 10^{31} y$, this in spite of the fact that there are a number of less massive particles to which it could decay. e.g.

$$p \to \pi^+ + \ldots, \quad p \to e^+ + \ldots$$

These observations motivated the introduction of the *baryon number* $\mathcal{B}$. The baryon number has the value $\mathcal{B} = 1$ for a single baryon and $\mathcal{B} = -1$ for an antibaryon. For all non-baryons, and hence for mesons, leptons and photons, have baryon number $\mathcal{B} = 0$. All members of an isospin multiplet necessarily have the same baryon number. The baryon number is an additive quantum number and hence for a nucleus involving A nucleons the baryon number will be just $\mathcal{B} = A$.

■ 2. Baryon Number Conservation

The observation that baryons do not appear to decay into non-baryons is consistent with the statement

The total baryon number in any closed system is conserved.

That statement summarises the experimental situation to date. Nevertheless there are many experiments being performed to establish limits on baryon conservation. Most of these involve trying to detect proton decay. The asymmetry between matter and antimatter is strongly suggestive of baryon non-conservation in the early universe.

■ 3. The Gell-Mann - Nishijima Charge Equation

Gell-Mann and Nishijima attempted to summarise the charges Q of hadrons in terms of the three quantum numbers I_z, $\mathcal{B}$, $\mathcal{S}$ and empirically found the relationship

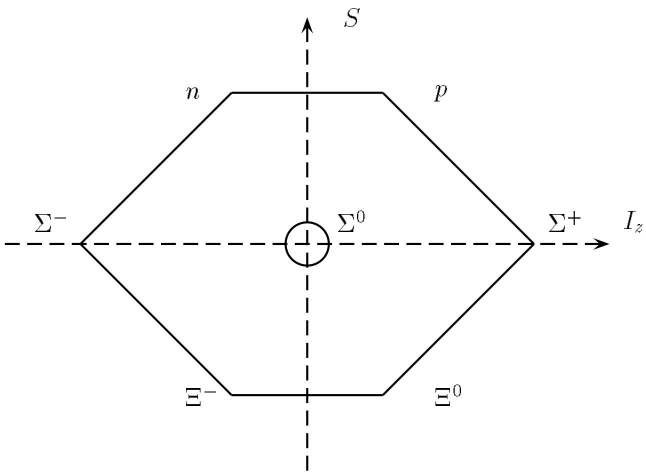
$$Q = I_z + \frac{\mathcal{B} + \mathcal{S}}{2} \tag{1}$$

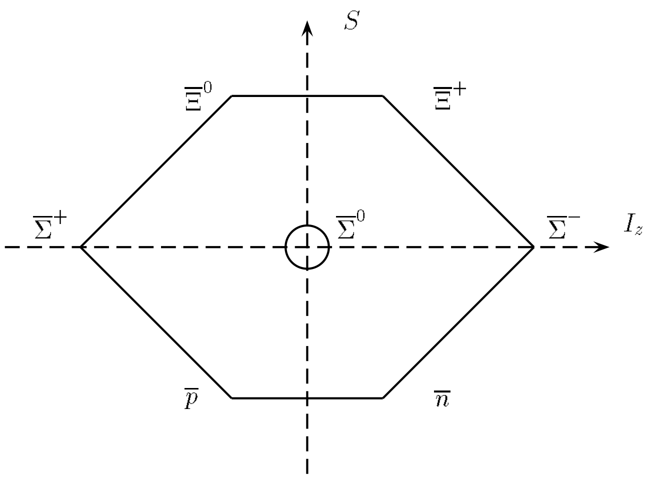
The appearance of $\mathcal{B}+\mathcal{S}$ suggested their combination be replaced by the equivalent *hypercharge* quantum number

$$\mathcal{Y} = \mathcal{B} + \mathcal{S} \tag{2}$$

■ 4. The Baryon Octet Plotted

The eight low mass spin $\frac{1}{2}$ baryons can be arranged into a highly suggestive pattern by plotting the strangeness quantum number $\mathcal{S}$ along a vertical axis versus the isospin projection I_z, along the horizontal axis, for each baryon as shown below for the baryons and antibaryons:





The Baryon Octet and ant-Octet

N.B. in going from particle to antiparticle we have

$$\mathcal{S} \to -\mathcal{S}, \quad I_z \to -I_z, \quad \mathcal{B} \to -\mathcal{B}, \quad Q \to -Q \tag{3}$$

Also note that particles of the same charge Q lie on sloping lines and can be expected to have similar electromagnetic properties.

■ 5. The low mass Spin 0 Kaons

As noted earlier, the discovery of the $\pi-$mesons was followed by the discovery of the $K-$ mesons, $K^{\pm}$ and K^0, $\bar{K}^0$, the *kaons*. The reaction

$$\pi^+ + p \to \Sigma^+ + K^+ \tag{4}$$

required that the K^+ be assigned strangeness $\mathcal{S} = +1$ and since K^- is its antiparticle partner it must have strangeness $\mathcal{S} = -1$.

The reaction

$$\pi^- + p \to \Lambda^0 + K^0 \tag{5}$$

is consistent with the assignment $\mathcal{S} = +1$ and hence the $\{K^0, K^+\}$ form an isospin doublet with $\mathcal{S} = +1$. The reactions

$$
\begin{aligned}
K^- + p &\rightarrow \bar{K}^0 + n \\
\pi^- + p &\rightarrow K^0 + \Lambda^0
\end{aligned} \tag{6}
$$

were consistent with K^0 having $\mathcal{S} = +1$ and $\bar{K}^0$ with $\mathcal{S} = -1$. Thus the $K-$mesons $K^-, \bar{K}^0$ also formed an isospin doublet but with $\mathcal{S} = -1$.

Thus the kaons may be grouped into two isospin doublets of opposite strangeness. The spin of the kaons were determined as $S = 0$, the same as found for the pions.

A further neutral meson η^0 with strangeness $\mathcal{S} = 0$ and spin $S = 0$ was found. Thus a group of eight low mass spinless mesons became known. Could these be organised into an approximate multiplet of eight particles as were the eight low mass baryons? ■ **6. The Eight Scalar Mesons**

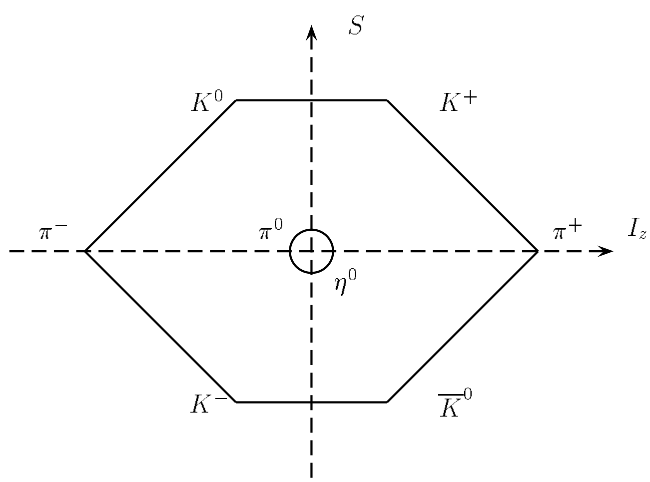
Let us now tabulate some of the properties of the eight mesons described so far

Properties of the Low Mass Scalar Mesons

Name	Mass	I	I_z	$\mathcal{S}$	Q
π^+	139.6		$+1$		$+e$
π^0	135.0	1	0	0	0
π^-	139.6		-1		$-e$
η^0	547.5	0	0	0	0
K^+	693.7		$\frac{1}{2}$		$+e$
		$\frac{1}{2}$		$+1$	
K^0	697.7		$-\frac{1}{2}$		0
$\bar{K}^0$	697.7		$\frac{1}{2}$		0
		$\frac{1}{2}$		-1	
K^-	693.7		$-\frac{1}{2}$		$-e$

■ 7. The Scalar Meson Octet

Let us plot the strangeness, $\mathcal{S}$, versus isospin projection, I_z as we did for the baryon octet as below:-

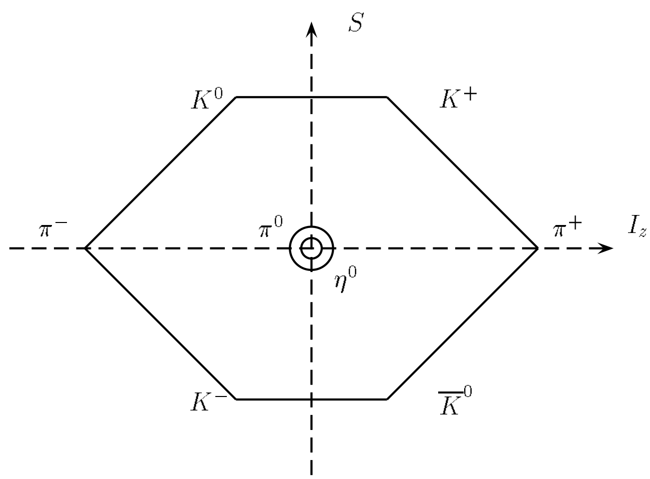


The Meson Octet

NB. In plotting the baryon octet the *particles* were placed in one octet and the *antiparticles* in a separate octet whereas for the scalar meson plot *both* appear in the same octet. This is consistent with the particle $\rightarrow$ antiparticle transformation given in Eq. (3), noting in particular that for *baryons* $\mathcal{B} \rightarrow -\mathcal{B}$ whereas for *mesons* $\mathcal{B} = 0$.

■ 8. Nonets of Mesons?

In addition to the neutral non-strange η^0 scalar meson a second neutral non-strange η'^0 meson of mass $958MeV$ was discovered. Including that in our plot leads to the *nonet* given below:-



The Meson Nonet

This raises the question "Do mesons come in groups of nine or should they be viewed as coming in octets and singlets?"

■ 9. Baryons of Spin $\frac{3}{2}$

The list of known baryons grew with the discovery of isospin multiplets of baryons of spin $\frac{3}{2}$. A non-strange isospin $I = \frac{3}{2}$ comprising a quartet of particles designated as

$$\Delta^-, \ \Delta^0, \ \Delta^+, \ \Delta^{++}$$

with mass $1230MeV$ to $1236MeV$ was found together with an isospin triplet with strangeness $\mathcal{S} = -1$

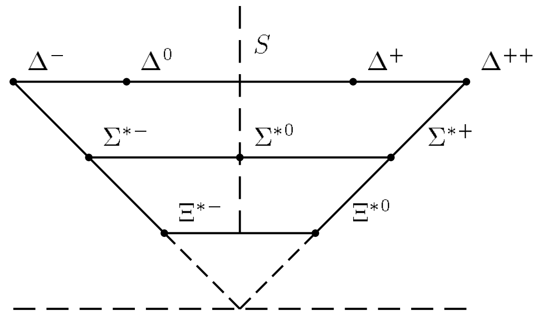
$$\Sigma^{*-}, \ \Sigma^{*0}, \ \Sigma^{*+}$$

with mass $\sim 1385MeV$ and an isospin doublet with strangeness $\mathcal{S} = -2$

$$\Xi^{*-}, \ \Xi^{*0}$$

with mass $1530MeV$.

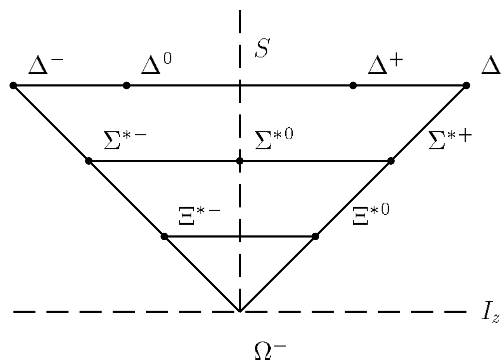
Note that the successive isospin multiplets differ in mass by $\sim 150MeV$. Furthermore plotting strangeness, $\mathcal{S}$, versus isospin projection, I_z, leads to a group of nine particles as shown on the next page:-

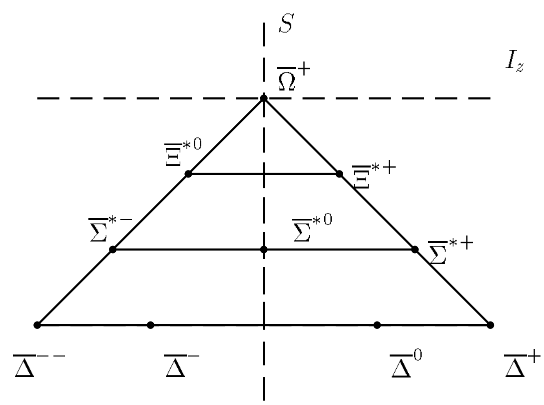


Nine Spin $\frac{3}{2}$ Baryons

■ 10. The Baryon Decuplet and Ω^-

What is the origin of the singlets and octets of mesons and the octets of baryons? Could there be patterns of baryons other than just octets as the spin $\frac{3}{2}$ baryons seemed to indicate? Gell-Mann and Ne'eman considered these questions in 1961 and came to the conclusion that the spin $\frac{3}{2}$ baryons should form a pattern involving ten particles, a *decuplet.* They concluded that there must be a missing baryon with spin $\frac{3}{2}$, isospin $I = 0$ and strangeness $\mathcal{S} = -3$ and a predicted mass of $\sim 1680MeV$ leading to the decuplet and anti-decuplets depicted below:-





The Baryon Spin $\frac{3}{2}$ Decuplet

Extensive searches for the missing Ω^- particle were undertaken and success was obtained in 1964 at Brookhaven National Laboratory. The mass of the Ω^- was found to be $1672 MeV$. This was the crowning result of particle physics and vindicated the "Eightfold Way" of Gell-Mann and Ne'eman. But, what is the origin of these patterns of one, eight and ten? Why octets and decuplets for baryons and singlets and octets for mesons? Can these patterns be built from a small set of particles more fundamental than the baryons and mesons? These questions are the subject of the next chapter.

Now this, O monks, is noble truth that leads to the cessation of pain: this is the noble Eightfold way: namely, right views, right intentions, right speech, right action, right living, right effort, right mindfulness, right concentration attributed to the Buddha.

Chapter Fifteen

Quarks and Hadrons

There is but one safe way to avoid mistakes; to do nothing...
This, however, may be the greatest mistake of all.
Albert Szent-Györgyi

SYNOPSIS

Can the low mass hadrons be represented in terms of a small set of particles? In other words - Are the hadrons composite particles? In this chapter we introduce the quark model of hadrons and see the origin of the singlets, octets and decuplets in terms of quarks.

■ 1. Partons

Scattering experiments involving the collision of electrons with protons probe the structure of the proton and in the late 1950's it was clear that the nucleon was composite, made up of *partons*, a phrase introduced by R. P. Feynman. What should we take as these fundamental entities or partons? Recall, mesons have integer spin and are *bosons* whereas the baryons have half-integer spin and are *fermions*. If we wish to build *both* mesons and baryons out of the same entities and if these entities are not themselves mesons or baryons then we should take the basic entities as fermions since we may combine fermions to form bosons but not vice versa*. Let us first see if we can represent the non-strange mesons and baryons in terms of some basic entities.

■ 2. Enter the u and d quarks

The simplest assumption is that the basic entities are spin $\frac{1}{2}$ fermions. The π^+ and π^- mesons form a particle-antiparticle pair and have spin 0. This suggests that the pions could be constructed by combining fermions with antifermions. The low mass baryons appear as a spin $\frac{1}{2}$ octet and a spin $\frac{3}{2}$ decuplet. This would be consistent with constructing baryons out of a triple products of fermions and antibaryons from triple products of the corresponding antifermions. The pions form an isospin triplet ($I = 1$) and since isospin projections are additive we can assume that the basic fermions form an isospin doublet ($I = \frac{1}{2}$). Following Gell-Mann, let us call this pair of fermions *quarks* and designate the $I_z = +\frac{1}{2}$ quark by the letter u (*the "up quark"*) and the $I_z = -\frac{1}{2}$ quark by the letter d (*the "down quark"*) with electric charges q_u and q_d respectively. The corresponding antiquarks $\bar{u}$ and $\bar{d}$ wil have opposite signs for their charges and isospin projections.

■ 3. Quark charges

The π^+ meson has $Q = +1$, $I = 1$, $S = 0$ and $I_z = +1$ which would be compatible with the assignment

$$\pi^+ \sim u\bar{d} \tag{1}$$

if we take

$$Q = 1 = q_u - q_d \tag{2}$$

The proton has $Q = +1$, $I = \frac{1}{2}$, $S = \frac{1}{2}$ and $I_z = +\frac{1}{2}$ which would be compatible with the assignment

$$p \sim uud \tag{3}$$

with

$$Q = 1 = 2q_u + q_d \tag{4}$$

Solving Eq. (2) and (4) gives the quark charges (in units of $+e$) as

$$q_u = \frac{2}{3} \qquad q_d = -\frac{1}{3} \tag{5}$$

Since mesons have baryon number $\mathcal{B} = 0$ and baryons $\mathcal{B} = 1$ it follows that the quarks must carry a baryonic charge of $\mathcal{B} = \frac{1}{3}$.

---

* If magnetic monopoles exist, and to date there is no evidence that they do, then it is possible to combine particles known as *dyons* (particles containing both electric and magnetic charge) to form fermions

■ 4. The Pions

If $\pi^+ \sim u\bar{d}$ then its antiparticle must be $\pi^- \sim \bar{u}d$. The neutral pion π^0 must be constructed as some linear combination of the quark-antiquark pairs $u\bar{u}$ and $d\bar{d}$**. We can form a second neutral meson state by taking a linear combination of the quark-antiquark pairs $u\bar{u}$ and $d\bar{d}$ that is orthogonal to that for the π^0.

The wavefunctions for the charged pions involve quark-antiquark pairs of *different* quarks whereas the neutral mesons involve quarks of the *same* type. Recall the charged pions have a meanlife of $2.6 \times 10^{-8}s$ whereas the neutral pion has the much shorter meanlife of $8.4 \times 10^{-16}s$. This remarkable difference can now be understood - the charged pions decay via the weak interaction whereas the neutral pion decays electromagnetically by quark-antiquark annihilation.

The basic ansatz for constructing mesons from quarks is:

Mesons are formed by coupling a quark to an antiquark

■ 5. The Non-Strange Baryons

The corresponding ansatz for constructing baryons is:

Baryons (Anti-baryons) are constructed out of a triplet of quarks (anti-quarks)

We have already noted that the proton may be built out of the quark configuration uud. For the neutron we need a quark configuration with isospin projection $I_z = -\frac{1}{2}$. This would be consistent with assigning the neutron as

$$n \sim udd$$

which corresponds to an uncharged baryon. However, note that the constituents of the neutron carry electric charge making it possible for the neutron to display a nett magnetic moment even though it is electrically neutral, indeed a simple quark model calculation leads to the result for the ratio of the proton to neutron magnetic moment as

$$\frac{\mu_p}{\mu_n}_{calc} = -\frac{3}{2} \qquad \frac{\mu_p}{\mu_n}_{expt} = -1.49$$

To construct the Δ baryon quartet requires a temporary suspension in belief in the Pauli Exclusion Principle (PEP). Consider the Δ^{++} baryon. It has an isospin $I = \frac{3}{2}$ and isospin projection $I_z = \frac{3}{2}$. This will only be possible if it is a coupling of three quarks in the configuration uuu. But the Δ baryons have spin $\frac{3}{2}$ and hence must be able to have four spin states $M_S = \pm\frac{1}{2}, \pm\frac{3}{2}$. However to create a three $u-$quark state with spin $\frac{3}{2}$ with spin projection $M_S = \frac{3}{2}$ requires that the spin states of each of the three $u-$quarks be $m_s = \frac{1}{2}$. Since the quarks in their groundstate carry no orbital angular momentum, unless there is a hitherto unknown quantum number capable of assuming three different values for a single quark, we have a clear violation of of the PEP! Let us write for the Δ^{++} in its maximal spin state

$$|\Delta^{++}\rangle = |\overset{+}{u}\overset{+}{u}\overset{+}{u}\rangle$$

The other members of the isospin quartet with maximum spin projection would be

$$|\Delta^{+}\rangle = |\overset{+}{u}\overset{+}{u}\overset{+}{d}\rangle, \quad |\Delta^{0}\rangle = |\overset{+}{u}\overset{+}{d}\overset{+}{d}\rangle, \quad |\Delta^{-}\rangle = |\overset{+}{d}\overset{+}{d}\overset{+}{d}\rangle$$

** in detail one forms the quark-antiquark wavefunctions

$$|\pi^+\rangle = -|u\bar{d}\rangle$$
$$|\pi^0\rangle = \frac{1}{\sqrt{2}}(|u\bar{u}\rangle - |d\bar{d}\rangle)$$
$$|\pi^-\rangle = |d\bar{u}\rangle$$

together with the orthogonal linear combination

$$|\eta^0\rangle = \frac{1}{\sqrt{2}}(|u\bar{u}\rangle + |d\bar{d}\rangle)$$

The various spin states of the Δ thus arise from the three-quark configurations

$$uuu \sim \Delta^{++}, \quad uud \sim \Delta^{+}, \quad udd \sim \Delta^{0}, \quad ddd \sim \Delta^{-}$$

and those of the antibaryon $\bar{\Delta}$ from

$$\bar{u}\bar{u}\bar{u} \sim \bar{\Delta}^{--}, \quad \bar{u}\bar{u}\bar{d} \sim \bar{\Delta}^{-}, \quad \bar{u}\bar{d}\bar{d} \sim \bar{\Delta}^{0}, \quad \bar{d}\bar{d}\bar{d} \sim \bar{\Delta}^{+}$$

■ 6. Strange Mesons

The $u, d-$quarks are unable to produce strange particles as they carry strangeness $\mathcal{S} = 0$. To produce strange particles we need to introduce a strange quark, which we will designate as the $s-$quark. Thus we have a set of three quarks and three antiquarks with the quantum numbers tabulated below:

quark	I	I_z	$\mathcal{S}$	Q
u		$\frac{1}{2}$		$\frac{2}{3}$
	$\frac{1}{2}$		0	
d		$-\frac{1}{2}$		$-\frac{1}{3}$
s	0	0	-1	$-\frac{1}{3}$

quark	I	I_z	$\mathcal{S}$	Q
$\bar{u}$		$-\frac{1}{2}$		$-\frac{2}{3}$
	$\frac{1}{2}$		0	
$\bar{d}$		$\frac{1}{2}$		$\frac{1}{3}$
$\bar{s}$	0	0	1	$\frac{1}{3}$

Combining the three quarks (u, d, s) with the three antiquarks $(\bar{u}, \bar{d}, \bar{s})$ leads to the formation of nine mesons with quantum numbers $(I_z, \mathcal{S}, Q)$ as below

Meson	I_z	$\mathcal{S}$	Q
$u\bar{d}$	1	0	1
$d\bar{u}$	-1	0	-1
$u\bar{u}$	0	0	0
$d\bar{d}$	0	0	0
$s\bar{s}$	0	0	0
$u\bar{s}$	$\frac{1}{2}$	1	1
$d\bar{s}$	$-\frac{1}{2}$	1	0
$s\bar{d}$	$\frac{1}{2}$	-1	0
$s\bar{u}$	$-\frac{1}{2}$	-1	-1

If we make an $\mathcal{S}$ versus I_z plot of the above quark-antiquark pairs we find six of them fall on the vertices of a hexagon with three at the centre of the hexagon. Comparison of the plot with that of the nine scalar mesons shows that the pair $d\bar{s}, u\bar{s}$ form an isospin doublet with strangeness $\mathcal{S} = 1$ and correspond to the meson isospin doublet K^0, K^+ while the pair $s\bar{u}, s\bar{d}$ correspond to the meson isospin doublet $K^-, \bar{K}^0$. $u\bar{d}$ has the quantum numbers of the π^+-meson while $d\bar{u}$ has those of the π^--meson. The π^0, η^0, η'^0 all have the same quantum numbers $I_3 = Q = \mathcal{S} = 0$ and must be formed from combinations of the states formed by the three quark-antiquark pairs $u\bar{u}, d\bar{d}, s\bar{s}$. Two of these go into forming members of the meson octet and one into forming a meson singlet. Thus we could write

$$\mathbf{3} \times \bar{\mathbf{3}} = \mathbf{1} + \mathbf{8}$$

Note that the pair $s\bar{s}$ involves strange quarks and yet the resulting state has $\mathcal{S} = 0$, such a state is said to have *hidden strangeness.*

■ 7. Baryons from quarks

Baryons are formed by coupling triplets of quarks, or in the case of antibaryons triplets of antiquarks. Recallin that $3 \times 3 \times 3$ we can expect to form a total of 27 baryons with the quantum numbers as shown below

quarks	I_3	$\mathcal{S}$	$\mathcal{Q}$	Number of Baryons
uuu	$\frac{3}{2}$	0	2	1
uud	$\frac{1}{2}$	0	1	3
udd	$-\frac{1}{2}$	0	0	3
ddd	$-\frac{3}{2}$	0	-1	1
uus	1	-1	1	3
dds	-1	-1	-1	3
uds	0	-1	0	6
uss	$\frac{1}{2}$	-2	0	3
dss	$-\frac{1}{2}$	-2	-1	3
sss	0	-3	-1	1

There is one baryon for each independent quark wavefunction. Thus for uds we can form six independent orthonormal sets of quark wavefunctions. It is instructive to make a plot of isospin projection, I_z, versus strangeness, $\mathcal{S}$, for the 27 states. The diagram can be resolved into a decuplet, two octets and a singlet corresponding to

$$\mathbf{3x3x3 = 1 + 8 + 8 + 10}$$

There is only one state of charge $Q = +2$, namely, that from the uuu quark configuration and it has the quantum numbers associated with the Δ^{++} spin $\frac{3}{2}$ baryon and thus the decuplet is associated with spin $\frac{3}{2}$ baryons. Of the two octets, it is possible to combine them to produce a baryon octet of spin $\frac{1}{2}$, note however that we have not considered in detail the antisymmetrization of our states.

■ 8. Masses of the Baryons in the Decuplet

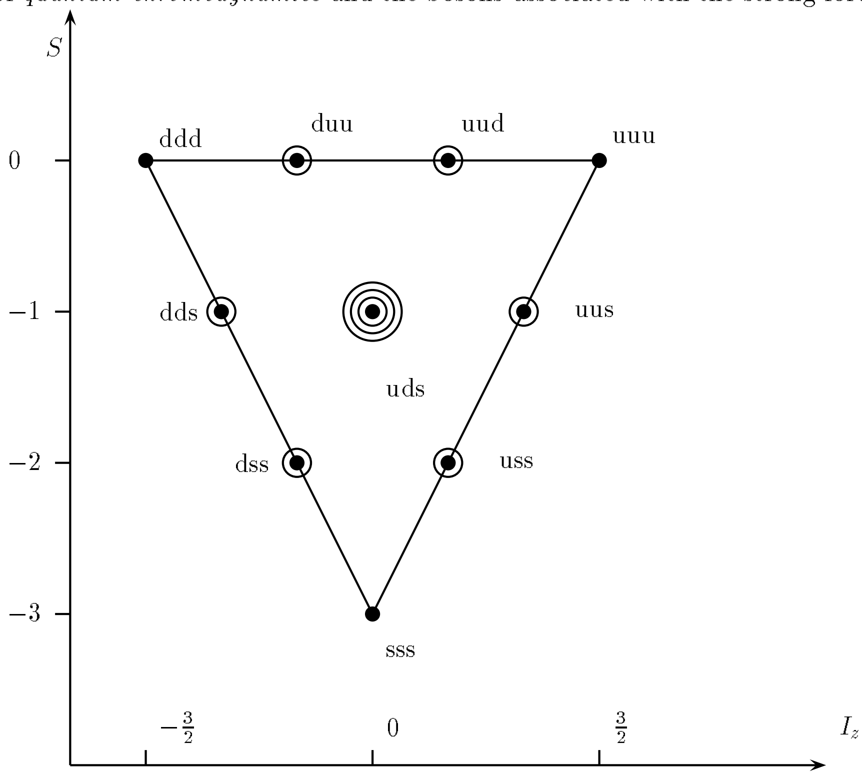
The baryons in the $S = \frac{3}{2}$ decuplet involve four isospin multiplets. The mass difference between successive multiplets is $\sim 150MeV$. This near equality of the inter isospin multiplets can be understood on the quark model by noting that the $u-, d-$quarks have the same isospin $I = \frac{1}{2}$ and hence can be expected to be of approximately the same mass. The strange $s-$quark has isospin $I = 0$ and should be more massive than the $u-, d-$quarks. Starting with the non-strange Δ isospin quartet one moves through successive isospin multiplets of the decuplet by successively adding a $s-$quark and hence to a good approximation we can anticipate the equality of the inter isospin multiplets.

■ 9. Coloured Quarks

We have all ready noted that as it stands our construction of the baryon decuplet violates the much loved Pauli Exclusion Principle. The PEP tells us that the only states allowed for identical fermions are those that are totally antisymmetric with respect to the interchange of any two particles. This requires, for example, in a system of electrons that no two electrons occur in the same state. Each one electron state is described by a set of quantum numbers and one cannot have two or more electrons having the same set of quantum numbers. Our Δ^{++} baryon in the state with $I_z = +\frac{3}{2}$ and $S_z = +\frac{3}{2}$ clearly involves three identical $u-$quarks which have the same set of quantum numbers. Antisymmetrisation for electrons is closely associated with the two-valuedness of the spin quantum number. However to construct a properly antisymmetrised Δ^{++} baryon would require a quantum number that can take on three different values so that each of the three $u-$quarks are distinguished. This three-valued quantum number has been called *colour* and figuratively can be labelled as red, (r), blue, (b), and green, (g). Without a further ansazt we would be led to a great increase in the number of possible baryons and mesons of various "colours". The ansazt is

The only particles that occur in nature are colourless

This means, for baryons the three colours must occur in equal admixtures. Recall the analogy - blue, red and green are primary colours and equal admixtures produce white. Antiquarks must have the corresponding anticolours so that the observable mesons are produced without colour. Here one might recall the analogy with hidden strangeness in the $s\bar{s}$ state. We shall note in a later time that while the introduction of colour saves the fermion statistics it also has other important consequences that lead to experimentally verifiable predictions and provides for the basis of a theory of the strong force in terms of *quantum chromodynamics* and the bosons associated with the strong force, the so-called *gluons*.



The 27 Baryons Plotted

Chapter Sixteen

More Quarks and Families

Tomorrow is not an extrapolation of today

SYNOPSIS

Experimental considerations led to the introduction of the charmed quark and the subsequent discovery of charmed mesons and baryons. Later, two additional quarks, *top* and *bottom* were introduced to complete the picture. The leptons and quarks appear in three families each containing 15 particles.

■ 1. The Charmed Quark

The introduction of the u, d and s quarks led to a greatly increased understanding of the baryons and mesons. In parallel to these developments had been the introduction of quantum chromodynamics with its coloured quarks. In the case of the u, d quarks this meant that they carried a total electric charge Q_q of

$$3 \times (q_u + q_d) = 3 \times (\frac{2}{3} - \frac{1}{3}) = +1 \tag{1}$$

The lepton e carried a charge -1 and one had for the collection of particles $\{u, d, e, \nu_e\}$ a total electric charge Q_f

$$Q_f = 3 \times (q_u + q_d) + q_e + q_{\nu_e} = 1 - 1 + 0 = 0 \tag{2}$$

Thus the total electric charge carried summed to zero. This balancing of the quark and lepton charges appeared in quantum chromodynamics as a necessary requirement for a consistent theory of quarks and leptons i.e.

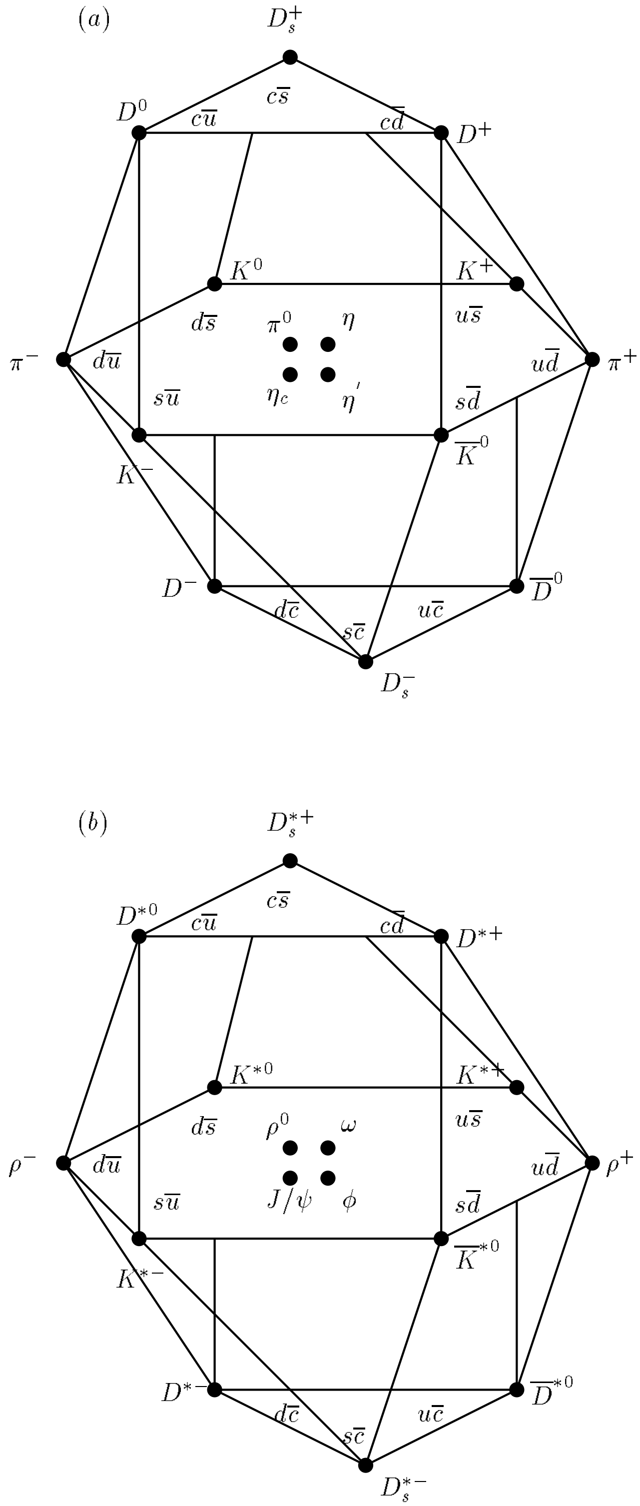
$$\sum (Q_{quark} + Q_{lepton}) = 0 \tag{3}$$

If we were to partner the strange quark s with the next lepton pair $\{\mu, \nu_\mu\}$ then Eq. (3) would no longer satisfied. To restore Eq. (3) it would be necessary to hypothesise the existence of an additional quark, c, that carried a charge $q_c = \frac{2}{3}$ which would be the partner of the strange quark and was termed the *charm* quark. The charmed quark had strangeness $\mathcal{S} = 0$ and carried its own quantum number $\mathcal{C} = 1$ termed the *charm* quantum number.

In the case of the u, d quark pair the d quark was slightly more massive than the u quark (Recall $m_{neutron} - m_{proton} \sim 0.5MeV$). The strange quark, s, was more massive than the u, d quarks and it was predicted that while the c quark was the partner of the s quark it would be rather more massive than the s quark. This implied that the resultant new mesons and baryons involving one, or more, charmed quarks would be significantly more massive than those involving just u, d, s quarks.

■ 2. Charmed Mesons

By 1974 the properties of the charmed mesons and baryons had been predicted in some detail. Seven charmed mesons should exist, three with charm $\mathcal{C} = +1$ $\{\bar{u}c, \bar{d}c, \bar{s}c\}$, three with charm $\mathcal{C} = -1$ $\{u\bar{c}, d\bar{c}, s\bar{c}\}$ and one with $\mathcal{C} = 0$ (*hidden charm*). The charmed quark, like the strange quark, had strong isopin $I = 0$. Previously we plotted out the quantum numbers of the mesons as a two-dimensional plot of $\mathcal{S}$ versus I_z. With the introduction of the charm quantum number we need to make a three-dimensional plot with $\mathcal{S}$ and I_z plotted in a plane, as before, with the charm quantum number $\mathcal{C}$ plotted on an axis perpendicular to the $\mathcal{S}, I_z$ plane to give the diagram overleaf.



SU(4) 16-plets for the (a) pseudoscalar and (b) vector mesons made of u, d, s, and c quarks.

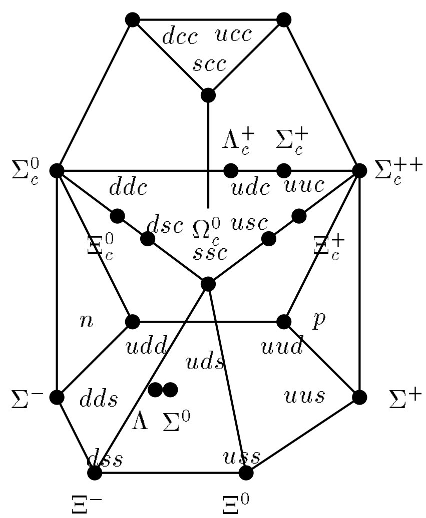
The nonets of the light mesons occupy the central planes, to which the $c\bar{c}$ states have been added. The neutral mesons at the centres of these planes are admixtures of $u\bar{u}$, $d\bar{d}$, $s\bar{s}$, and $c\bar{c}$ states

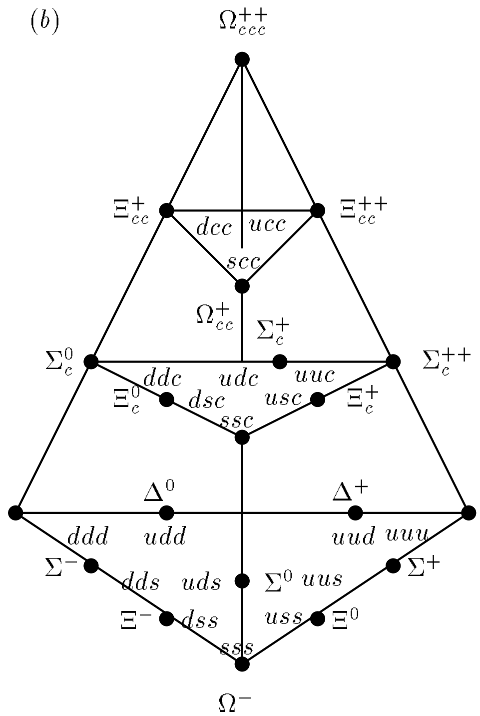
Note the appearance at the centre of the figure of a single meson $c\bar{c}$ with hidden charm $\mathcal{C} = 0$ this was the so-called $J\Psi$ particle discovered simultaneously at Brookhaven and Stanford in 1974. Subsequently all the particles indicated in the meson picture have been discovered and their decays established in considerable detail.

■ 3. Charmed Baryons

Additional baryons are possible if the charmed quark is introduced. Unlike mesons, no hidden charm states can arise and hence every baryon involving one or more charmed quarks has a definite $\mathcal{C}$ number. The baryons with $\mathcal{C} = 0$ involve just the u, d, s quarks and give rise to the normal baryon octet and decuplet when a $\mathcal{S}, I_z$ plot is made. If $\mathcal{C}$ is plotted perpendicular to the $\mathcal{S}, I_z$ plane we obtain the two figures given below:

(a)





SU(4) multiplets of baryons made of u, d, s, and c quarks. (a) The 20-plet with an SU(3) octet. (b) The 20-plet with an SU(3) decuplet.

The production of the first charmed baryons was accomplished at the Fermilab accelerator.

■ 4. More Quarks

The discovery of charmed particles strengthened the believe among physicists that there was a pair of quarks for each family, or generation, of leptons. Thus the u, d quarks were linked to the e, ν_e leptons, s, cquarks to the μ, ν_μ leptons and ... . Was there a further pair of quarks b, t linked to the τ, ν_τ leptons? Furthermore it had been suggested that a total of six quarks could lead to a solution to the $\mathcal{CP}$ puzzle associated with the neutral kaon mesons $K^0, \bar{K}^0$ which violated $\mathcal{CP}$ conservation, and if $\mathcal{CPT}$ invariance held would imply non-invariance with respect to $\mathcal{T}$ (time-reversal). This hypothesis was greatly strengthened in 1977 with the production, again at Fermilab, of the $\Upsilon-$meson involving the expected b quark (or *bottom* quark). The bottom quark had an electric charge of $-\frac{1}{3}$ and carried its own quantum number B (not to be confused with the baryon number $\mathcal{B}$) with the other quantum numbers $I, \mathcal{S}, \mathcal{C}$ all zero. To plot out the quantum numbers of the mesons and baryons involving the quantum numbers $\mathcal{S}, I_z, \mathcal{C}, B$ requires four-dimensional graph paper which seems unobtainable in Poland, and elsewhere. Nevertheless one can, of course, list the quartets of quantum numbers.

The discovery of mesons and baryons involving the bottom quark greatly stimulated attempts to find experimental evidence for the existence of a t quark (*top* quark). Tantalising hints of its existence developed in 1994 and unequivocal evidence for a meson $t\bar{t}$ was made public in early 1995. The top quark has an electric charge of $+\frac{2}{3}$ and carries its own quantum number T. A plot of the five quantum numbers $\mathcal{S}, I_z, \mathcal{C}, B, T$ requires five-dimensional graph paper!

Thus a highly satisfying picture of three generations of quarks and leptons is emerging.

■ 5. The First Generation

The existence of three generations of quarks and leptons is surprising. All the particles required for ordinary matter seem to involve just the members of the first generation. Let us look at the first generation in a little more detail. It involves the u, d quarks and the e, ν_e leptons. According to quantum chromodynamics the u, d quarks each come in three "colours" making six coloured quarks while the leptons e, ν_e do not carry colour. The leptons and quarks are all spin $\frac{1}{2}$ fermions and should come in both left-handed, L, and right-handed, R, forms. However, recall the neutrino associated with the electron occurs only in the left-handed form ν_L.

Physicists have attempted to bring a semblance of order to the particles of a given generation by grouping together the left-handed particles into a single family (the right-handed particles form a similar family). The left-handed electron, e_L, and the left-handed neutrino, ν_L, are very light particles and are grouped into a colourless *weak* isospin doublet, $I^{wk} = \frac{1}{2}$,

$$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \tag{4}$$

with ν_L having $I_z^{wk} = +\frac{1}{2}$.

The left-handed quarks u_L, d_L are likewise grouped together as a coloured weak isospin doublet

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \tag{5}$$

The left-handed antiquarks, designated as d_L^c and u_L^c, are anticoloured weak isospin singlets. Likewise the left-handed positron appears as a weak isospin singlet designated as e_L^c (We have attached a superscript c to indicated the conjugate particle). We are led to a group of 15 left-handed particles, remembering that the quarks and antiquarks come in three colours,

$$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad u_L^c, \quad d_L^c, \quad e_L^c \tag{6}$$

■ 6. The SU_5 Picture

Can one group the fifteen particles of Eq. (6) into a more meaningful order? Each particle is associated with a definite electric charge, which in turn may be related to a weak hypercharge quantum number Y^{wk}, a definite weak isospin I^{wk} and a colour state. Observable particles always occur as colour singlets (i.e. they involve equal admixtures of the three colour quantum numbers. Physicists try to represent these properties in terms of mathematical symmetry groups. For example the quantum theory of angular momentum is founded upon the rotational invariance of free space and the mathematical group is SO_3, the group of rotations in three-dimensions. In the case of half-integer angular momentum the *covering* group of SO_3, the special unitary group in two-dimensions, SU_2, is used. Every rotation in three dimensions is equivalent to a special unitary transformation in two dimensions. The group SU_2 is the group appropriate to the weak isospin and we designate it as $SU_2^{I^{wk}}$. It is associated with *weak* interactions. The weak hypercharge Y^{wk} is associated with the electric charge of particles and hence with *electromagnetic* interactions. The charge is quantised. Electromagnetic interactions are associated with photons and the relevant group is the group of unitary transformations in one-dimension, U_1^{em}. Thus electromagnetic/weak (or *electroweak* interactions are associated with a very special group structure written as

$$U_1^{em} \times SU_2^{I^{wk}} \tag{7}$$

Quarks are strongly interacting particles and, as noted earlier, each have three possible colour states. The three colour states may be associated with the special unitary group SU_3^c known as the colour group or the group of strong interactions. Thus we might try to describe both quarks and leptons with a group structure

$$U_1^{em} \times SU_2^{I^{wk}} \times SU_3^c \tag{8}$$

Such a structure nevertheless treats the electromagnetic, weak and strong interactions separately. The physicists dream is to be able to unite all three interactions in some **G**rand **U**nified **T**heory (**GUT**) and perhaps ultimately to unite such a theory with the fourth interaction, that of gravity, to produce a **T**heory **O**f **E**verything (**TOE**).

The first attempt at producing a GUT involved trying to find a single group that contained within itself the entire structure of Eq. (8). This lead to the SU_5 picture that attempted to combine the quarks and leptons into particular *representations* of the special unitary group SU_5. This led to the classification of the first generation of leptons and quarks as in the table below which we will return to in the next chapter.

SU_5	fermion	I	I_z	Y	Q	color
$\bar{\mathbf{5}}$	ν_L	$\frac{1}{2}$	$\frac{1}{2}$	-1	0	$\mathbf{1}^c$
	e_L		$-\frac{1}{2}$		-1	
	d_L^c	0	0	$\frac{2}{3}$	$\frac{1}{3}$	$\bar{\mathbf{3}}^c$
$\mathbf{10}$	u_L	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\mathbf{3}^c$
	d_L		$-\frac{1}{2}$		$-\frac{1}{3}$	
	u_L^c	0	0	$-\frac{4}{3}$	$-\frac{2}{3}$	$\bar{\mathbf{3}}^c$
	e_L^c	0	0	2	1	$\mathbf{1}^c$

What God hath put asunder no man shall join together Wolfgang Pauli

Chapter Seventeen

Pictures of Forces

Anything that happens, happens.
Anything that, in happening, causes something else to happens, causes something else to happen.
Anything that, in happening, causes itself to happen again, happens again.
It doesn't necessarily do it in chronological order, though.
— Douglas Adams *Mostly Harmless* London: Heinemann (1992)

SYNOPSIS

So far we have discussed the properties and classification of the fermionic leptons and quarks and the construction of hadrons from quarks. At the conclusion of the previous chapter we sketched one attempt to produce a **GUT**. This had the effect of bringing together the quarks and leptons into families of 15 particles in three generations. But what of the interactions that bind the quarks and antiquarks together in the formation of hadrons? Before tackling that problem we shall first consider the various pictures of the forces of nature. Virtual reality is nothing new to physicists and much more exciting than the computer variety!

■ 1. The Three Generations

In the previous chapter we ended up with the first generation of fermions being classified as in the table below:

SU_5	fermion	I	I_z	Y	Q	colour
$\bar{\mathbf{5}}$	ν_L	$\frac{1}{2}$	$\frac{1}{2}$	-1	0	$\mathbf{1}^c$
	e_L		$-\frac{1}{2}$		-1	
	d_L^c	0	0	$\frac{2}{3}$	$\frac{1}{3}$	$\bar{\mathbf{3}}^c$
$\mathbf{10}$	u_L	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\mathbf{3}^c$
	d_L		$-\frac{1}{2}$		$-\frac{1}{3}$	
	u_L^c	0	0	$-\frac{4}{3}$	$-\frac{2}{3}$	$\bar{\mathbf{3}}^c$
	e_L^c	0	0	2	1	$\mathbf{1}^c$

The three colours of the quarks are designated by $\mathbf{3}^c$ and for the antiquarks by $\bar{\mathbf{3}}^c$ where the c indicates colour. The leptons are associated with colourless states designated as $\mathbf{1}^c$[1]. The above table involves the first family, or generation, of 15 quarks and leptons. The further two families of 15 have exactly the same structure as in the table given for the first family. It is an experimental observation that there are three generations. In a more complete theory one would like to know why there are three families (and whether there are only three families). This problem remains to be resolved.

[1] Technically, $\mathbf{3}^c$, $\bar{\mathbf{3}}^c$ and $\mathbf{1}^c$ correspond to the dimensions of certain irreducible representations of the colour group SU_3^c.

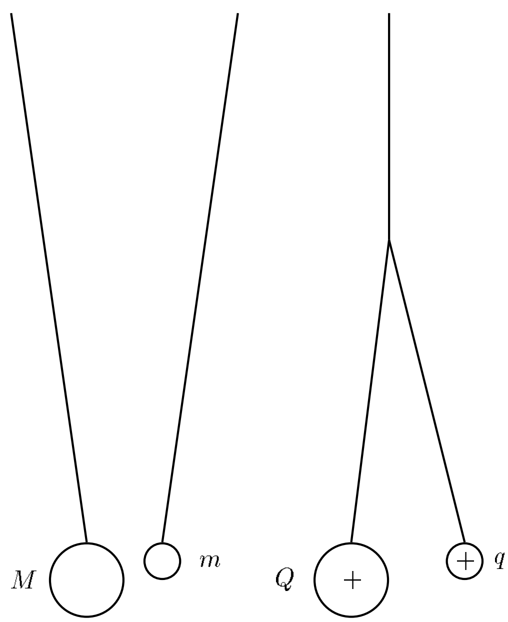
■ 2. Quantum ElectroDynamics (QED)

Quantum electrodynamics is the theory of the quantization of electromagnetic interactions and has been an extraordinarily successful theory. Many properties of the hydrogen atom can be calculated to virtually experimental accuracy. In such calculations the masses of the electron and proton do not come out of the theory but must be put in by hand, i.e. the experimentally measured masses are used in the calculations. Electromagnetic interactions are associated with a massless particle, the photon (γ), which is a spin 1 boson, but before going further we need to consider the concept of force.

■ 3. The Concept of Force

What is a force? There are several ways of looking at this question.

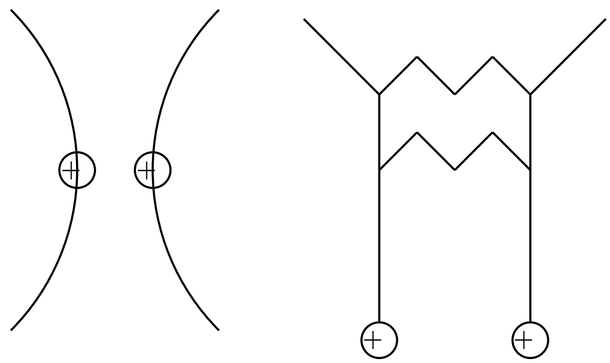
1. **Action-at-a-distance Picture**
   Here we might think of a mass at one point "feels" the gravitational force of a mass at some other point, or a charge at one point being repelled by a charge at some other point, or the earth's magnetic core forcing a magnetic compass needle to turn around.
2. **Classical Field Picture**
   In the classical field picture the action-at-a-distance picture is changed to a two-stage process. The mass, charge or magnet creates a field whether or not there is another particle to feel (or probe) it. In this picture we have the notion of an electric field, gravitational field or a magnetic field in empty space. Placing a mass, charge, or magnet at a point results in it sensing the field present locally and experiences a force proportional to the field and to its own mass, charge or magnetic strength.



$$F = \frac{GMm}{r^2} \qquad\qquad F = \frac{Qq}{4\pi\epsilon_0 r^2}$$

The classical field picture originated with Faraday and became a quantitative theory of electromagnetism through Maxwell. The introduction of Einstein's theory of relativity in a sense completed the development of the classical field picture.

3. **The Quantum Picture**
   Classically two charged particles repel each other smoothly leading to smooth trajectories. In the quantum picture the process occurs in jerks (or jumps)[2]



(a). Classical picture (b). Quantum picture

In the quantum picture the particle interaction between fermions (and/or antifermions) takes place via the exchange of bosons. The repulsion of two particles by a force can be pictured as involving a series of jumps in which the exchanged boson is successively absorbed by one particle and then re-emitted and reabsorbed by the second particle. Since momentum is conserved in the exchange process the direction of the particles will change in jerks.

Electromagnetic forces arise because charged particles can exchange photons (γ). The photon is the spin 1 boson of quantum electrodynamics. If you charge up a gold leaf electroscope the leaves move apart because they are vigorously exchanging photons! In the quantum picture you can get attractive forces as well as repulsive forces.

Q1. Why can't we "see" the light particles? Why can't we block them with an opaque material such as cardboard? These are very difficult questions which will take us time to answer.

Gravitational forces presumably[3]arise by the exchange of a boson known as the *graviton*.

■ 4. Virtual Particles

The exchange particles involved in forces are often referred to as *virtual* particles. These particles exist only to mediate the forces and have some distinctive properties not shared by the *"real"* particles that trigger photomultipliers and such like detectors. The virtual particles are operating at a submicroscopic level and as such are permitted to go faster, or slower, than the speed of light c but on the average go at the speed c. They are in a sense free to choose their speed. This is one of the fundamental differences

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[2] *jerk* n. sharp sudden pull or twist. (Oxford dictionary)

[3] I say *presumably* as the graviton remains undetected, due in part from the extreme weakness of the gravitational force

between quantum theory and classical (or relativistic) mechanics. Things allowed in classical physics are allowed in quantum theory but things not allowed in classical physics are often allowed in quantum theory. Quantum theory associates a probability with every possible action and the overall effect is found by summing all the possibilities. In most, but not all, situations the effects that are classically allowed are the most important. Over long observation times (recall a typical nuclear time can be as short as $10^{-23}s$!) the particles are found to obey special relativity and travel at speed c.

Thanks to the uncertainty principle, violation of energy conservation is possible over a short time interval subject to

$$\Delta E \dot{\Delta} t \sim h \tag{1}$$

Thus in the exchange of a particle by absorption, or emission, momentum is conserved but energy need not be conserved except in the overall process. While the virtual particle is travelling in the space between the original particles a mismatch ΔE in the total energy at that time can be allowed provided the virtual particle lives only a finite time Δt given by Eq. (1).

■ 5. The Cardboard Question and Refraction of Light

We have seen that in quantum theory energy nonconservation is allowed for a finite time but always on a payback basis. When two charges interact with one another through a sheet of cardboard the exchange photons may be stopped in the cardboard for a while - they may be absorbed by the atoms or molecules in the cardboard. But energy is conserved in the long time and those atoms or molecules must release their energy by re-emitting the photons. The net result is that the photons are slowed down. The slowing down is a dynamic process comprising short hops and stops till the photon reaches the other side. This is also the quantum picture of the refraction of light[4]. We can see now that the physics of refraction is closely related to the physics of optical absorption. Understanding the commonly observed "bending of a stick in water" is a very subtle problem requiring the subtleties of quantum theory and relativity to be discussed with any degree of honesty and completeness.

■ 6. Electrons as Garbage Carriers

Our electron can no longer be thought of as a single entity but, like all charged particles, is surrounded by a cloud of virtual photons, each living as long a time as allowed by the uncertainty principle. On average, more energy is required to produce the cloud of virtual particles - even temporarily. Thus in a very real sense the charged particle is surrounded by not only virtual photons but also a host of other virtual particles since, on the pay back basis, the photons can be momentarily disappearing as a particle-antiparticle pair which subsequently annihilates giving back the photon. The effect is to affect the mass of the electron. We cannot directly observe this mass shift since we never see an electron without charge - it would not be an electron any longer. We can however measure related quantities where the virtual particles contribute such as to the magnetic moment of the electron[5].

■ 7. The Casimir Effect

The physicist's concept of the vacuum is nothing like the classical picture of of nothingness. Rather the vacuum is seen as a place of great activity - a boiling soup of electron-positron pairs, virtual in nature, according to QED - nothing forbids it now so it happens[6]. Clearly such imaginative ideas need testing! Can we predict any new effects from such ideas? If not then our theory is sterile. One such prediction was made by the Dutch physicist, Hendrik Casimir. Casimir suggested that one could change the vacuum by enclosing it in metal plates. These will short out those electromagnetic effects which occur at wavelengths of the order of the plate spacing, or greater. Their contribution to the total boiling dynamics is lost. As a consequence their contribution to the temporary and net energy shifts should be lost and the energy of the vacuum changed. If I pull the plates apart I am effectively putting back into the vacuum. This energy has to come from somewhere - namely the pull I gave to the plates. Thus I am led to the conclusion that two metal plates in a vacuum experience a force of attraction. This force is proportional to Planck's constant, h, (Thus it is a quantum effect and very small!) and inversely

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[4] Indeed Newton had the basic idea, if not the picture, that in entering a material his corpuscles slowed down.

[5] Technically these effects are associated by physicists as *mass renormalisation* effects

[6] Recall the Aristotlean statement *"Nature abhors a vacuum"*

proportional to c^2 (Thus it involves relativity and c^{-2} is very small!) and to the separation of the plates. The total effect is extremely small but measurable! The verification of the Casimer Effect is but one of a number of experiments supporting the physicists conception of the vacuum.

■ 8. The Lamb Shift

1947 saw the completion of two major experiments - Polykarp Kusch's accurate measurement of the magnetic moment of the electron and the measurement of the so-called *Lamb shift* by Willis Lamb and Robert Retherford. Both experiments were in disagreement with the Dirac theory of the electron and required the development of a new approach to QED - at the hands of Bethe, Dyson, Schwinger, Tomonaga and Feynman. Both results involved the effects of virtual photons.

We have seen that virtual photons interact so as to give the electron a jerky motion (called by physicists *zitterbewegung*). This means that for an electron in an atom the energy of interaction between the electron and the nucleus can be different due to the zitterbewegung and the difference in the energy shift is different for different electron orbits. Furthermore there is an additional contribution from the tendency of electron-positron pairs produced in the vacuum to be polarised by the nucleus. As a consequence the attraction of the electron to the nucleus is changed with the vacuum acting like a dielectric, polarising and shielding the nuclear charge from the orbiting electrons. Dirac's relativistic electron theory applied to the hydrogen atom was remarkably precise. However, the two orbits $2S_{\frac{1}{2}}$, $2P_{\frac{1}{2}}$ in the Dirac picture are degenerate, i.e. they have the same energy. The importance of the Lamb-Retherford experiment was its demonstration of a small splitting, or energy separation, between those two orbits now measured as

$$\Delta E_{expt} = 1057.862 MHz \pm 0.02 MHz \tag{2a}$$

and is commonly termed the *Lamb Shift*. Modern QED calculations give a theoretical splitting of

$$\Delta E_{th} = 1057.864 MHz \pm 0.014 MHz \tag{2b}$$

The splitting is extremely small and requires microwave spectroscopy to detect it. The apparent discrepancy between theory and experiment is properly due to the need to estimate the effect of an infinite number of ever decreasing terms left out in the calculation. Nevertheless the agreement is remarkable and constitutes further evidence of the physicists conception of the vacuum and its vacuum induced fluctuations.

■ 9. The Magnetic Moment of the Electron

One of the triumphs of the Dirac relativistic electron theory was its direct calculation of the magnetic moment of the electron as $g = 2$ exactly! agreeing with experiment until Polykarp Kusch's measurement in 1947. Writing

$$g = 2(1 + a) \tag{2}$$

modern experiments give

$$a_{expt} = (1.158\ 652\ 193\ \ \pm 0.000\ 000\ 004) \times 10^{-3} \tag{3a}$$

or

$$g = 2.0023293\ldots \tag{3b}$$

QED calculations taking into account the effects of virtual photons give

$$a_{th} = (1.159\ 652\ 197\ \ \pm 0.000\ 000\ 076) \times 10^{-3} \tag{3c}$$

The agreement to nine decimal places is truly remarkable and a further indication of the validity of QED.

■ 10. To the Future!

The calculations and experiments just alluded to all relate to the hydrogen atom. Nowadays it is possible to strip heavy atoms of all their electrons but one to leave a hydrogenic atom with a nuclear charge $+Ze$ where Z is the atomic number. Already ions such as U^{91+} have been made. These studies will, I believe, be prominent in the physics of the next century since the QED corrections all increase strongly with increasing atomic number and thus we should eventually get stronger tests of QED than those presently available. Helium like ions will be able to test our understanding of two-electron interactions. Of course there are complications - one must understand in more detail the nature of the interaction of electrons with the nucleus as in highly ionised atoms the electron orbits are much closer to the nucleus and again we must turn to a better understanding of the relationship of atomic physics to nuclear physics and in turn of particle physics. Note however, that now one cannot expect to understanding cosmology without particle physics AND vice versa. In the next chapter we return to the question of the forces of nature.

■ Physics Research is Too Expensive!

Before jumping to that conclusion don't forget the world budget for defence exceeds US$1,000,0000,000,000 per annum! or if you like US$2.71Bn per day or ∼US$100,000,000 per hour. The world health cost of smoking is estimated by the World Bank at ∼US$200Bn per annum. The cost of physics research is insignificant compared with these figures.

Chapter Eighteen

What Glues the Particles Together? I.

Knowledge for the sake of understanding,
not merely to prevail, that is the essence of our being
None can define its limits, or set its ultimate boundaries
— Vannevar Bush *Science is Not Enough* (1967)

Lecture 18. What Glues the Particles Together? I.

SYNOPSIS

Classifying the quarks and leptons into three families or generations. (Probably the word *generations* is more apt than *families* as generations imply sequential relationships between families which is closer to the physicists dream of unification) is to some extent botanical or taxinomic. As Fermi remarked "If I knew there would be so many particles I would have been a botanist". To make a physical picture we need to introduce interactions - the glue that sticks the particles together to form matter as we know it. QED provides the prototype for a general theory of the forces of nature. We associate interactions with the exchange of bosonic particles.

■ 1. The Bosons of Weak Interactions

The exchanged particle for QED is the massless photon γ. The exchanged particles giving rise to the weak interaction must also be bosons of spin 1. There is however a fundamental difference - electromagnetic interactions have an infinite range whereas the weak force has a very short range $< 10^{-17}m$. Yukawa's early attempt to explain nuclear forces in terms of the exchange of mesons involved short range forces but with a range ~ 100 greater than for the weak force leading to the idea that the exchange particles associated with weak interactions must be much more massive than Yukawa's mesons. By 1967 predictions of the mass of the exchanged particles (often referred to as *weakons*) were already being made. Furthermore, unlike for QED, three types bosons are involved in weak interactions, a triplet W^+, Z^0, W^-. The $W^\pm$ are a particle-antiparticle pair so on the basis of $\mathcal{CPT}$ invariance should have the same mass while the neutral Z^0 is its own antiparticle and can have a mass different from that of the $W^\pm$. (Technically the weakon triplet is associated with the adjoint representation of the weak isospin group $SU_2^{I^{wk}}$ forming an isospin triplet with three charge states corresponding to $W^\pm$ and Z^0 with zero weak hypercharge).

The masses were estimated to be $\sim 80GeV$. These particles were produced at CERN in 1983 and their masses measured as

$$M_{W^\pm} = 80.22GeV \qquad M_{Z^0} = 91.87GeV$$

The particles were produced by observing very energetic proton-antiproton collisions to produce

$$p^+ + p^- \rightarrow W^+ + W^-$$

or

$$p^+ + p^- \rightarrow Z^0 + ..,$$

Neither weakon can be observed directly but is identified by its subsequent decays such as, for example,

$$\begin{aligned} W^+ \rightarrow & \mu^+ + \nu_\mu \\ & e^+ + \nu_e \\ & \tau^+ + \nu_\tau \\ & hadrons \end{aligned}$$

and

$$\begin{aligned} Z^0 \rightarrow & e^+ + e^- \\ & \mu^+ + \mu^- \\ & \tau^+ + \tau^- \\ & hadrons \end{aligned}$$

■ 2. Electroweak Unification?

Prior to Maxwell, electric and magnetic forces were seen as separate, and unrelated forces. With Maxwell's introduction of an electromagnetic theory, and even more so with Einstein's theory of special relativity, electric and magnetic forces became united into a single force - the electromagnetic force and thus arose the first example of unification of seemingly different forces. Could the weak force and the electromagnetic force be woven into a single electroweak theory that encompassed both? At first this seems most unlikely, while the photon is a massless particle the weakons are very massive particles.

Why is the photon massless and the weakons so massive? Answering the first part of the question seems to lead to a massive contradiction in the second part of the question. A clue to the first part is that electromagnetism is associated with the, apparently strict, conservation of electric charge. An "exact" conservation law must involve an "exact" symmetry. This symmetry is of a different type to that associated with such things as energy and momentum conservation which arise from specific continuous symmetries of spacetime such as displacements and rotations. Conservation of charge is not of that type.

■ 3.Gauge Symmetry

There is a category of symmetries that are distinctly different from those symmetries we have earlier considered - these are the so-called *gauge transformations*. In physics we represent fields in terms of complex wave functions $\phi(x)$ that depend in some way upon the coordinates of spacetime x. (Here I use just x to stand for the four spacetime variables commonly given as, say, x, y, z, t placing space and time on an equal footing.) Gauge symmetries are very much associated with the freedom of choice the physicist has in defining $\phi(x)$. While the choice of $\phi(x)$ is usually severely constrained by physical considerations one expects to be free to specify the overall phase of the wavefunction without changing the physics deduced from the wave function. That is, the physics (Technically this means that the Lagrangian constructed from the functions $\chi(x)$ is invariant under a phase transformation) should be *invariant* under a transformation of the wavefunction such that

$$\chi(x) \rightarrow e^{i\alpha}\chi(x) \tag{1}$$

where α is called *the phase angle* (If $\alpha = 360°$ we have $e^{i\alpha} = +1$ or if $\alpha = 180°$ then $e^{i\alpha} = -1$ etc.) If the phase angle α is *not* dependent on the spacetime point x then the invariance is said to be a *global gauge* invariance.

Often the gauge transformation will be written in an equivalent, though more explicit form as

$$\chi(x) \rightarrow e^{iQ\lambda}\chi(x) \tag{2}$$

where now Q is the *charge* associated with the field $\chi(x)$ and λ is a real parameter.

In the case of electromagnetism Q may be rewritten as eQ in terms of the fundamental charge e. Whereas for global invariance the parameter λ is the same at all spacetime point relaxing that restriction and allowing the parameter λ to depend on the spacetime point x leads to *local gauge invariance.* i.e.

$$\lambda = \lambda(x) \tag{3}$$

This means that the phase angle is now arbitrary at every point x and local gauge invariance means that *the phase difference between two different spacetime points is not measurable.*

■ 4. Gauge Symmetry and Charge Conservation

The assumption of local gauge invariance immediately leads to conservation of the charge Q. Thus turning things around, the existence of charge conservation implies that it is experimentally not possible to measure a phase difference at two spacetime points - that is the "impossible experiment" associated with charge conservation. Technically the phase rotations form a one-dimensional Abelian unitary group $U(1)$. Lepton and baryon conservation are likewise associated with local gauge invariance and the "charges" are termed leptonic and baryonic charges respectively. The principle of local gauge invariance is very powerful - in the case of electromagnetism the principle alone suffices to derive the full theory of electromagnetism as contained in Maxwell's equations.

■ 5. So How do Particles Acquire Mass?

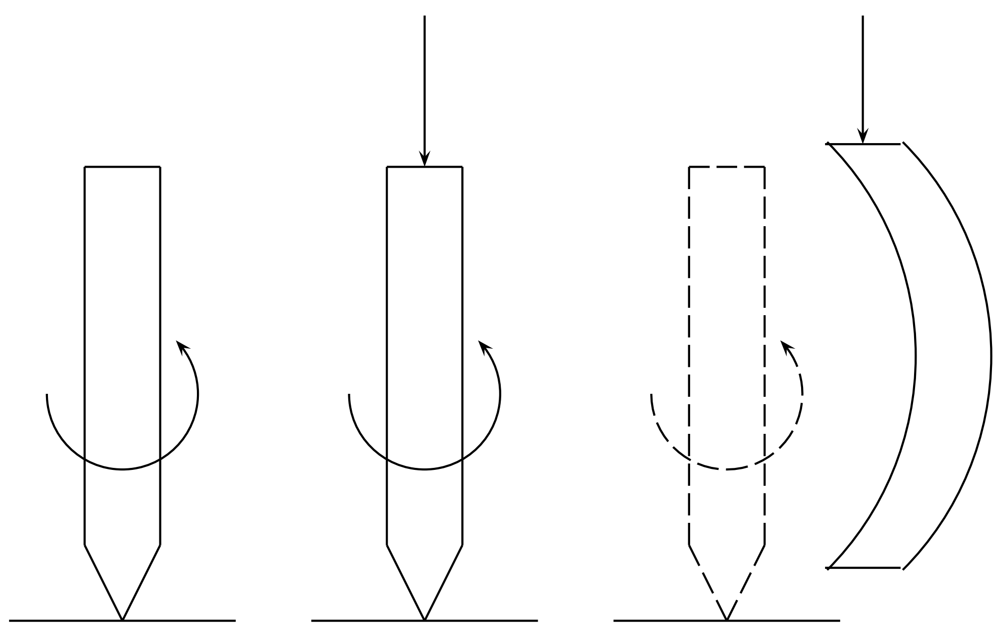
Local gauge invariance is associated with *gauge particles*, in the case of electromagnetism the gauge particle is the photon. Such invariance forces the gauge particle to be massless as is indeed the case for electromagnetism. Given the success of local gauge theory in electromagnetism we are tempted to carry over the formalism to weak interactions and therein lies a stumbling block - the photon is massless but the gauge particles of the weak interaction are massive. Initially it appears that *all* gauge particles should be massless so how do the weakons $W^{\pm}$, Z^0 acquire mass?

It was realised in 1954 by Yang and Mills, and by Shaw (YMS), that the gauge theory of electromagnetism associated with the unitary group $U(1)$ of phase rotations could be extended to more general gauge groups such as the $SU(2)$ group of the weak isospin BUT this gave the gauge bosons as massless (Technically YMS extended the Abelian $U(1)$ gauge theory of electromagnetism to non-Abelian gauge theories involving several gauge bosons. Thus $SU(2)$ became the non-Abelian gauge group of the weak interactions and involved three massless gauge bosons $W^{\pm}$, Z^0.). This was the dilemma facing physicists in the late 1950's and early 1960's. The solution was to be found once again in the mysteries of the vacuum and so-called *spontaneous symmetry breaking.*

■ 6. Spontaneous Symmetry Breaking

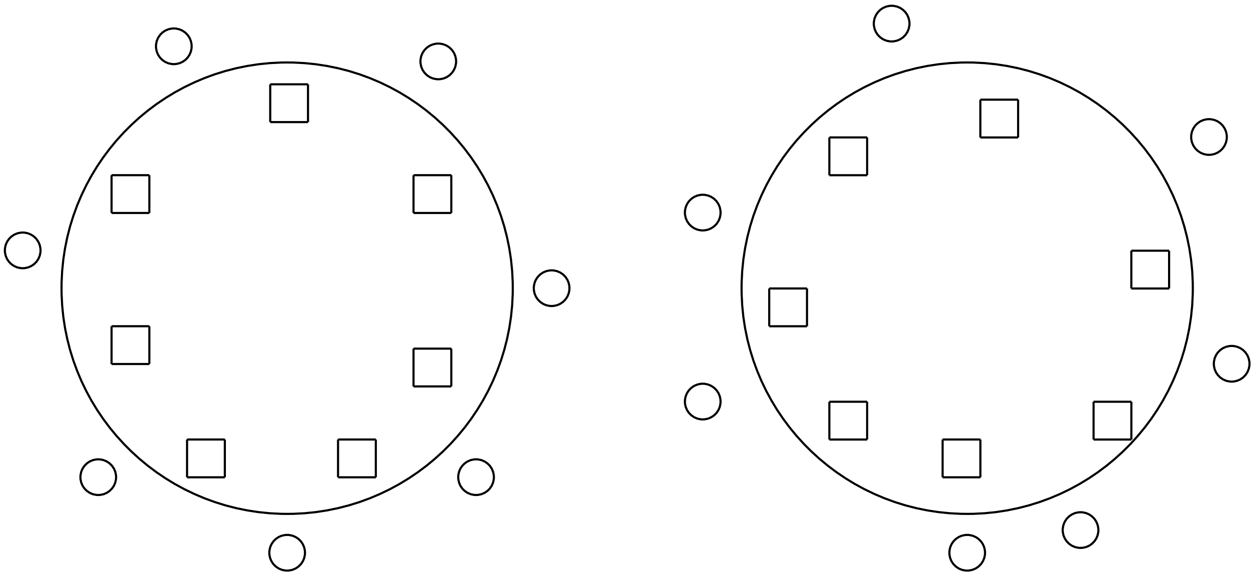
The concept of spontaneously broken symmetry originated in solid state physics and illustrates the importance of physicists being aware of work outside their immediate field of endeavour. It also shows the inter-dependency of different areas of physics and why we cannot say that research in one area is irrelevant to other areas. Furthermore ideas generated in some seemingly exotic area of physics may be of crucial significance in a more practical areas of physics and vice versa. The concept of gauge invariance started with Maxwell in electromagnetism theory and with Einstein in general relativity.

There are many apparent paradoxes in symmetry. A pencil appears to have symmetry with respect to arbitrary rotations about the axis passing through its length and hence the group of rotations in two-dimensional space, $SO(2)$. If I apply a gentle force pushing down on the top of the pencil the rotational symmetry remains. I gradually increase the downward force and I suddenly see the symmetry broken - the pencil bends in an unpredictable direction and the symmetry, and perhaps the pencil, is destroyed.



(a) Rotational Symmetry (b) Downward Force Applied (c) Spontaneously Broken Symmetry

As another example, imagine a group of diners sitting around a circular table. Each diner observes a table napkin to their right and to their left. The situation is completely symmetrical. One diner decides to pick up a table napkin on his/her left this breaks the symmetry and forces the diner to the left to also pick up the napkin to the left and the breaking of the symmetry, starting with an arbitrary choice, is propagated right round the table.



(a) Left-Right Symmetry (b) Broken Left-Right Symmetry

As yet another example imagine a donkey placed between two piles of hay. The situation has left-right symmetry and the donkey starves unless it chooses to break the symmetry with a decision to eat from the left or the right - the decision is arbitrary but the symmetry is broken.

Before continuing two brief diversions.

■ 7. Effective Mass in Solid State Physics

A further clue to the resolution of the mass problem came from solid state physics. In free space the electron has a definite measurable mass m_e. A solid contains a crystal lattice of positively charged ions and an electron wandering through the lattice is attracted to it. If we attempt to measure the electron's mass in such an environment we can find it has an effective mass m_e^* that may be many times m_e. In a sense the electron has *gained* mass by interaction with its environment.

■ 8. Spontaneous Symmetry Breaking and the Heisenberg Ferromagnet

A Heisenberg ferromagnet is an infinite lattice in which every lattice site is occupied by a spin $\frac{1}{2}$ magnetic dipole. The spins on neighbouring sites interact with a spin-spin interaction. This interaction is rotationally invariant and hence we might expect it would be impossible for the spins to assume any preferred direction in the lattice but that is not the case! The interaction while rotationally invariant results in the spins becoming aligned to create a ground state of the system that is NOT rotationally invariant! It is energetically favourable for the spins to all line up in one direction but the direction chosen, in the absence of external magnetic fields, is random. Nothing in the underlying physics of electromagnetism can lead to a prediction of the direction taken up by the aligned spins. This example emphasises the observation that while a given interaction may exhibit a particular symmetry the symmetry of the ground state may be different from that of the driving interaction. (One might compare the case of the Jahn-Teller effect in crystals where the crystal may lower its symmetry by a distortion that reduces the degeneracy of the ground state and in the process lowers the energy of the ground state.)

■ 9. Spin, Massless and Massive Particles

It is a characteristic of massless particles such as photons, gravitons and presumably the massless gauge bosons to possess just two spin (or helicity) states, either the spin is parallel to the momentum (positive helicity) or the spin is antiparallel to the momentum (negative helicity). Thus the photon having spin $S = 1$ has just two helicity states corresponding to the spin projections onto the momentum of $M_S = \pm 1$ and presumably for the graviton which has spin $S = 2$, just $M_S = \pm 2$. A massive particle (In physics a *massive* particle is a particle that has non-zero mass as opposed to a *massless* particle.) of spin S has a total of $2S + 1$ values of the spin projection M_S. Thus a massive particle of spin $S = 1$ should have three spin (or helicity) states. Compared with a massless $S = 1$ particle it has a state with $M_S = 0$ (zero helicity). How can we start with a massless spin $S = 1$ which has just two helicity states and end up with a massive particle with three spin states? Two discoveries were to shed light on this problem - the introduction of Goldstone bosons and the Higgs bosons. Again the transfer of results from the theory of the solid state played a crucial role in the solution that has led to what is now referred to as The Standard Model.

■ 10. The Goldstone Boson

In 1960/61 Nambu and Goldstone independently studied the problem of broken global symmetries and showed that a spontaneously broken continuous symmetry gives rise to massless spin $S = 0$ bosons. This seemed, at the time, very surprising for it was felt that surely such a particle would have already been discovered. Goldstone, Salam and Weinberg studied the problem further and concluded that whenever a symmetry like isospin or strangeness is spontaneously broken Goldstone bosons must occur. (Even worse they would remain massless to all orders of perturbation theory.) It thus appeared that the solution of the problem of mass was further away than ever.

■ 11. The Higgs Mechanism to the Rescue

A way out of the Goldstone boson dilemma developed in 1964 with the work of Higgs, Kibble and others. It was shown that the massless gauge bosons of the Yang-Mills-Shaw theory and the Goldstone bosons could be simultaneously avoided via what has become known as the *Higgs mechanism.* (It is perhaps linguistically strange to use the word *mechanism* with its overtones of 18/19th century mechanistic views. Rather Higgs developed a *procedure, method or recipe* for generating masses from interaction of particles with the Higgs field.) The Higgs method is a particularly beautiful specialisation of Lagrangian field theory which while technically transparent to anyone familiar with field theory is a challenge to adequately describe in words, a task that I shall attempt in the next chapter. The Higgs method was to supply the missing piece in understanding weak interactions and in the hands of Ward, Salam and Weinberg was to lead to a unified theory of weak and electromagnetic interactions, the electroweak theory. The weak interaction cannot supply the "glue" to bind particles together - it, like gravity, is much too weak. However, much of the methodolgy developed in establishing the electroweak theory was to play an important role in developing a theory of the "glue", but that must await another chapter. All these developments were also to impact strongly on the relationship between particle physics and cosmology as particle physics allows us to look back in time much further than any telescope.

■ Serendipity *serendipity:*faculty of making happy discoveries by accident. *Oxford Dictionary*

> "... I remember travelling back to London on an American Airforce transport flight. ... I could not sleep. I kept reflecting on why Nature should violate left-right symmetry in weak interactions. ...While crossing over the Atlantic, came back to me a deeply perceptive question "The photon mass is zero because of Maxwell's principle of gauge symmetry for electromagnetism ... why is the neutrino mass zero?" ... But during that comfortless night the answer came. The analog for the neutrino of the gauge symmetry for the photon existed: it had to do with the masslessness of the neutrino. ...Nature had the choice of an aesthetically satisfying but a left-right symmetry violating theory, with a neutrino which travels exactly with the speed of light; or alternatively a theory where left-right symmetry is preserved, but the neutrino has a tiny mass - some ten thousand times smaller than the mass of the electron" *Abdus Salam,*Nobel Prize Address

> "... At some point in the fall of 1967, I think while driving to my office at MIT, it occurred to me that I had been applying the right ideas to the wrong problem. It is not the ρ meson that is

massless: it is the photon. And its partner is not the A1, but the massive intermediate bosons, which since the time of Yukawa had been suspected to be the mediators of the weak interactions. The weak and electromagnetic interactions could be described in a uniform way in terms of an exact but spontaneously broken gauge theory. And this theory would be renormalizable like quantum electrodynamics because it is gauge invariant like quantum electrodynamics." *Steven Weinberg,*1979 Nobel Prize Address

Chapter Nineteen

What Glues the Particles Together? II.

"... The goddess of learning is fabled to have sprung full grown from the brain of Zeus, but it is seldom that a scientific conception is born in its final form, or owns a single parent. More often it is the product of a series of minds, each in turn modifying the ideas of those that came before, and providing material for those that come after"

*G. P. Thomson,*1937 Nobel Prize Address

SYNOPSIS

The Yang-Mills-Shaw gauge theories led to the embarrassing result that ALL gauge bosons were massless. While in agreement with QED where the gauge boson, the photon, was indeed massless that could not be the case for the gauge bosons of the weak interaction. A way out was sought in spontaneously broken global symmetries but this introduced massless spin zero Goldstone bosons. The Higgs mechanism was to supply the way out of the double dilemma of massless gauge bosons and Goldstone bosons and to create a method for generating massive bosons.

■ 1. The Central Problem of 1964

The central problem of 1964 in the theory of weak interactions and attempts to produce a unified theory of weak and electromagnetic interactions was how to generate masses for the weak interaction bosons while at the same time keeping the photon as a massless gauge boson and to somehow get rid of the unwanted Goldstone bosons. Is it possible to somehow spontaneously break the gauge symmetry and avoid the Goldstone bosons?

■ 2. The Higgs Solution

The way started to become clear with Peter Higgs seminal paper *Broken Symmetries and the Masses of Gauge Bosons* {Phys. Rev. Lett. **13**, 508-9 (1964)}. Higgs noted parallel developments in the theory of superconductivity. In particular *"Like the "superconductor theories" these gauge theories have suffered from a zero mass difficulty: The gauge principle appears to guarantee that the associated vector*[7]*field quanta are massless, ... But the only known massless vector boson is the photon; the existing evidence suggests that all other vector bosons must be massive.* Higgs followed with a more detailed paper entitled *" Spontaneous Symmetry Breakdown without Massless Bosons"* {Phys. Rev. **145**, 1156-63 (1966)}

Higgs realised that breaking global symmetries would not do the trick. What was needed was to spontaneously break down the local gauge symmetry. Higgs needed to find a way to cause his gauge vector bosons to interact with some field to break the local gauge symmetry and for his gauge bosons to acquire mass. To do this he needed to start with field equations that satisfied the symmetry of the gauge group but with a vacuum state that did not preserve the local gauge symmetry leading to a spontaneous breaking of local gauge invariance.

■ 3. Higgs Model Calculation

Higgs set up a model or prototype calculation in which he imagined the vacuum was permeated by a pair of scalar fields, a Higgs doublet[8]. His massless gauge vector bosons, represented by two real functions, interacted with the scalar fields. After spontaneous breakdown of the local symmetry he obtained an equation[9]describing an interacting massive scalar field, represented by a real function, together with a massive vector field, represented by three real functions. There was no embarrassing Goldstone boson, it had disappeared in the spontaneous symmetry breakdown being transformed to supply the extra degree of freedom required for a massive vector boson[10]. The massive scalar field has as its quanta a spinless

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[7] *Scalar fields* are associated with spin zero bosons whereas *vector fields* are associated with spin one bosons.

[8] Strictly speaking a *complex scalar field* involving two *real functions.*

[9] The Lagrangian of the system.

[10] Recall that a massless vector boson has spin one but only two spin states, $M_S = \pm 1$ - there is

neutral boson, commonly called the *Higgs Boson*.

Higgs' model showed how massless gauge bosons could gain mass by interacting with the Higgs field which is assumed to permeate all space. The larger the interaction of a particle with the Higgs field the more mass the particles appear to have. Higgs' model gave a mechanism for generating masses but while he recognised its relevance to the theory of weak interactions it was to be up to Weinberg and Salam to apply Higgs' model to create a realistic unification of electromagnetic and weak interactions.

■ 4. The Salam-Weinberg Electroweak Theory

QED was associated with the massless vector gauge boson, the photon (γ), while it appeared that the gauge bosons of the weak interaction involved three massless vector bosons ($W^{\pm}, Z^0$), the weakons. Thus electromagnetism, unified with weak interactions, must be associated with four massless gauge vector bosons. The problem was to be able to give mass to the weakons while leaving the photon massless. The gauge group was designated $U_1 \times SU_2$ with the photon belonging to U_1 and the weakons to SU_2.

The first clue was the Yang-Mills-Shaw theory for breaking the symmetry of non-Abelian groups such as SU_2 albeit with the generation of massless Goldstone bosons. Higgs spontaneous symmetry breaking gave the second clue as it avoided the Goldstone bosons and showed how gauge bosons could gain mass. Salam and Weinberg, independently, applied the Higgs mechanism to break the $U_1 \times SU_2$ gauge symmetry down to the U_1 gauge symmetry of electromagnetism leaving the photon massless while the weakons gained mass as a result of their coupling to the Higgs field.

■ 5. Predictions of the Electroweak Theory

The electroweak theory , in its simplest form, is characterised by a single parameter $\sin^2\theta_W$, where θ_W is the mixing angle between the bare Z^0 and γ. It was possible to express the masses of the weakons $W^{\pm}, Z^0$ in terms of $\sin^2\theta_W$. In 1967 no accelerators existed capable of creating the weakons. The ratio of the masses of the charged ($W^{\pm}$) to the neutral (Z^0) weakons was predicted to be

$$M_Z = \frac{M_{W^{\pm}}}{\cos\theta_W} \tag{1}$$

However, every prediction at that stage depended upon a knowledge of the parameter $\sin^2\theta_W$.

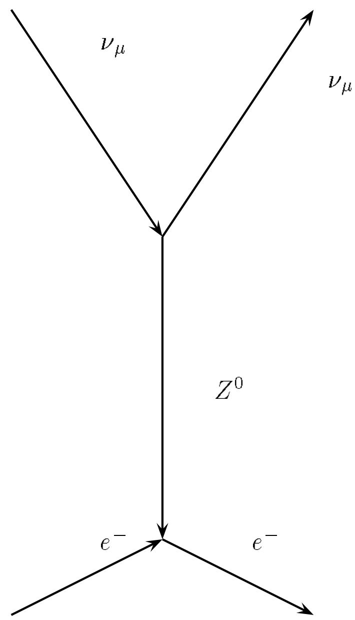
The introduction of the neutral Z^0 boson led to the prediction of so-called *neutral currents* which essentially meant it should be possible to scatter neutrinos off electrons with no change in charge as for example in reactions such as

$$\nu_\mu + e^- \rightarrow \nu_\mu + e^- \tag{2a}$$

$$\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^- \tag{2b}$$

which were mediated by the exchange of a Z^0 boson as illustrated in the diagram overleaf:-

no zero helicity, or longitudinal state with $M_S = 0$. In Higgs model the Goldstone boson supplies this missing state.



The process $\nu_\mu + e^- \to \nu_\mu + e^-$

Such a neutral current was observed at CERN in 1973. The cross-section for such a neutral current process can be expressed in terms of the parameter $\sin^2\theta_W$ and hence led to the first experimental determination

$$\sin^2\theta_W = 0.25^{+0.07}_{-0.05} \tag{3}$$

Modern measurements give a value of

$$\sin^2\theta_W = 0.2319 \pm 0.0005 \tag{3$'$}$$

The observation of neutral currents was the first verifiable result of the electroweak theory and was the basis of the Nobel Prize award to Glashow, Salam and Weinberg in 1979, even prior to the observation of the $W^\pm$ and Z^0 bosons at CERN in 1983. The measurement of $\sin^2\theta_W$ and subsequent improvements in the measurement then allowed, by 1982 the mass predictions

$$M_W = 83.0^{+3.0}_{-2.8} GeV \tag{4a}$$

$$M_Z = 93.8^{+2.5}_{-2.4} GeV \tag{4b}$$

which may be compared with the currently known masses of

$$M_W = 80.22 \pm 0.26 GeV \tag{4a'}$$

$$M_Z = 91.187 \pm 0.007 GeV \tag{4b'}$$

■ 6. Mass Generation for Fermions

So far we have presented just the boson mass generation by spontaneous symmetry breaking via the coupling of the gauge bosons to the Higgs field. Fermions also couple to the Higgs field giving mass to the quarks and charged leptons. As yet we do not have the ability to make detailed predictions of the masses of fermions and I will not pursue this topic at this moment.

■ 7. Parity Violation in Atoms

As noted the weak interaction between fermions is mediated by $W^{\pm}, Z^0$ bosons[11]. The electroweak theory leads to the prediction of **P**arity **N**on-**C**onserving (**PNC**) interactions between electrons and the fermionic constituents of nucleons (the quarks in the form of protons) leading to the admixing of states of opposite parity leading in turn to handed-ness of emitted photons. These effects are extremely small - almost at the limits of experimental detectability. However the effects increase rapidly with increasing atomic number Z. Thus in thallium and bismuth the effects are some six orders of magnitude greater than for hydrogen.

In the much studied case of Bi_{83} atom the effect of PNC is to give a very small mixing of the opposite parity states of the $6p^3$ and $6p^2 7s$ electron configurations. This has as a consequence a predicted rotation of the plane of polarization of the emitted photons of $\sim 10^{-7} radians$. Basically one places a column of bismuth vapour between two nearly crossed polarizers and looks for changes in intensity $\sim 1 : 10^{-3}$ corresponding to a predicted rotation angle of $\sim 3 \times 10^{-7} radians$.

Experiments for both bismuth and thallium have yielded, albeit with great experimental difficulties, results in agreement with the electroweak theory.

■ 8. The Higgs Boson - The Missing Link

The electroweak theory has been extraordinarily successful with many confirmed predictions. Combined with quantum chromdynamics, the theory of strong interactions, we have what has become known as the Standard Model (**SM**). The SM has stood up to, and survived, many tests. Many of the features of the SM are well tested, however, there remains an important missing link, the Higgs boson, quanta of the Higgs field. The SM gives no prediction as to the mass of this enigmatic particle though some bounds have been predicted. The Higgs boson is a neutral spin zero particle making its detection especially difficult. The discovery of the Higgs boson would supply the missing link and will be one of the key objectives of the European Large Hadron Collider. The Higgs boson is unlikely to be observed in this century.

■ 9. Unification of Forces

The electroweak theory appears to successfully unify the weak and electromagnetic forces into a coherent unified theory. At our energy scale these two forces appear very distinctive. Each force is characterised by a coupling constant which is a measure of the strength of the respective forces. However, the coupling constants are energy dependent and at some $10^{16} GeV$ are expected to be of the same magnitude. The electroweak theory has the weak angle, θ_W, as a parameter which cannot be determined from within the theory. The next step must be to attempt to unify the electroweak theory with the theory of strong interactions. We then find testable predictions of the size of θ_W. There appears to be a price to pay in such a grand unification - leptoquarks that threaten to decay protons! This must await the next chapter.

■ Some Quotable Physics

> *... which suddenly seemed to change the role of Goldstone bosons from that of unwanted intruders to that of welcome friends*, S. Weinberg.
>
> *We now realize, with special clarity, how much in error are those theorists who believe theory comes inductively from experience. Even the great Newton could not free himself from this error.*, A. Einstein
>
> *Pure logical thinking cannot yield us any knowledge of the empirical world; all knowledge of reality starts from experience and ends in it.*, A. Einstein

You may notice some contradiction in the above Einstein quotations!

[11] Of course other interactions between fermions are possible such as the strong interaction between quarks. However, these do not violate parity conservation.

Chapter Twenty

What Glues the Particles Together? III.

"I quickly found out the difference between a Fellowship and a job. The former pays at the beginning of the month, the latter at the end."
*Cecilia Payne-Gaposchkin*1900-1979

SYNOPSIS

We have seen how particles may gain mass, and worrisome Goldstone bosons may disappear but that does not tell us what glues the particles together. That is the task of QCD and its eight gluons! This leads to a theory of strong interactions. Can the theory of strong interactions be unified with the unified electroweak theory? Are there observable, or testable, predictions from such a Grand Unified Theory?

■ 1. Freedom and Slavery

Yukawa's theory of mesons gave the first picture of the strong interactions that bind protons and neutrons in the nucleus. We now know that such a theory cannot explain the very much stronger forces that bind together the quarks in neutrons and protons. Yukawa's mesons, viewed as bound states of quark-antiquark pairs, indeed give a picture of nuclear forces but at a more fundamental level one expects to be able to discuss both nuclear forces and the forces between quarks, and antiquarks, in terms of quark-quark and quark-antiquark interactions. The nuclear forces in some sense can be regarded as the analogue of van der Waals forces between molecules with the weak van der Waals forces appearing as a left over part of the much stronger Coulomb force.

The force that binds quarks and antiquarks appears to be of a quite different nature to those encountered in electroweak theory and any successful theory must descibe this difference. No free quark has ever been observed in spite of numerous attempts. There is good evidence to suggest that mesons are formed as bound states of quark-antiquark pairs and baryons as triplets of quarks. However, no bound states involving a pair of quarks has been observed, or indeed of four or five quarks.

Probing protons by energetic beams of electrons or photons seems the indicate the proton is made up of point-like quarks. Within the hadrons the quarks appear to be asymptotically free. When they are close together they enjoy freedom but as they move apart their freedom becomes slavery. There appears no way for them to escape and enjoy the freedom experienced by the leptons. If we supply sufficient energy to set free the quark we observe a quark-antiquark pair, or meson.

■ 2. The Necessity of Colour

What is the origin of the strongly attractive forces that bind $\bar{q}q$? Based upon our experience with electroweak theory it is tempting to associate the strong forces with the exchange of some spin one bosons that "glue" the quarks together in forming hadrons, the "gluons". What properties must we attribute to these gluons? We have already argued that the quarks and antiquarks must carry a colour quantum number capable of assuming three values or colours. This was necessary to save the Pauli exclusion principle in building a quark model of the Δ^{++} spin $\frac{3}{2}$ baryon from three u quarks.

There is a considerable body of other evidence supporting the introduction of colour. For example, it is possible to calculate exactly the decay rate for the decay $\pi^0 \to 2\gamma$ but the calculation requires summing over all the quarks coupling to the π^0 and this is proportional to the number of colour degrees of freedom N_c attributed to the quarks. The observed decay rate can only be satisfied if $N_c = 3$.

Furthermore, experimentalists have measured the products , hadrons and mesons, produced in electron-positron collisions. the cross-section ratio

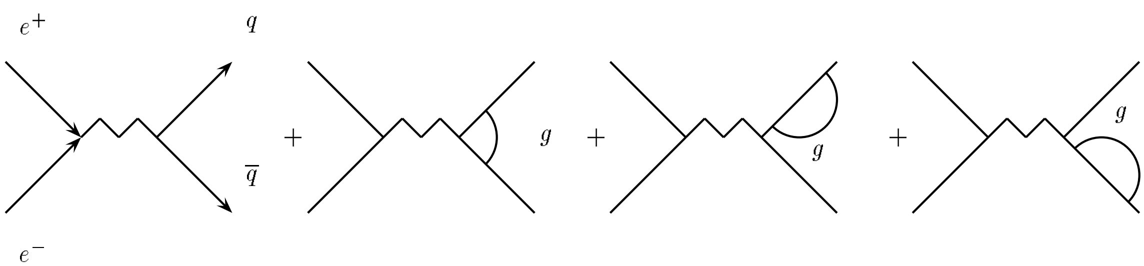
$$\frac{\sigma(e^+e^- \to \ hadrons)}{\sigma(e^+e^- \to \ \sum_i \bar{q}_i q_i)} \propto N_c$$

Again we find the measured ratios being compatible with $N_c = 3$.

■ 3. Coloured Gluons and QuantumChromoDynamics (QCD)

Quantum chromodynamics is the current theory of strong interactions and we expect it to remain an integral part of any more comprehensive theory that may be developed in the future in the same sense as Newton's mechanics remains as an important approximation to Einstein's more comprehensive theory of relativity. QCD is a gauge theory of the strong interactions and has at its heart a mathematical structure based on the special unitary group SU_3^c where the superscript reminds us that this is the symmetry group associated with the colour group. We have already met the group SU_3 in our discussion of the symmetries of baryons and mesons. There we encountered objects fitting into singlets, octets and decuplets, most notably the baryon octet of Gell-Mann's Eight-Fold-Way. However, the group of QCD, while mathematically the same as for the hadrons is applied in a very different manner. In QCD it is a group of gauge transformations.

Central to QCD are an octet of spin one massless gauge bosons termed gluons. These eight gluons carry colour but not quantum numbers such as charge Q, isospin I, or hypercharge Y. These latter three quantum numbers are associated with the electroweak theory of $SU_2^{I_{wk}} \times U_1^{Y_{wk}}$ In QCD the quarks interact through the eight bosons, the gluons which are the gauge bosons of QCD. The gluons play the role of the photon of electromagnetic interactions but with a fundamentally difference - the photon does not interact with itself whereas gluons do! This feature makes QCD very different from QED! This is seen in part in the diagrams shown below. The first diagram illustrates the electromagnetic interaction contribution while the succeeding diagrams show the effects of gluon interactions.



Some interaction diagrams for $e^+ + e^- \rightarrow \bar{q}q$

It is a feature of QCD that the only observable "free" particles are those corresponding to colour singlets (i.e. states in which the colour is effectively washed out). Thus the gluons and quarks can only appear in objects without colour such as hadrons. It is impossible to construct colourless states from quark configurations where the number of quarks is a multiple of three or the number of quarks and antiquarks are equal. Colourless combinations of gluons could form observable "glueballs" though no such objects appear to have been found. A similar situation appears to hold for exotic quark configurations such as q^6 etc. The primary evidence for the existence of gluons come from predictions based upon QCD such as in the appearance jets in high energy collisions of particles and in the self-consistent picture of strong interactions that emerges from QCD. It is a feature of QCD that the force between quarks increases linearly with separation, a feature quite distinct from other known forces.

■ 4. The Standard Model (SM)

The electroweak theory combined with QCD is commonly referred to as the Standard Model (SM) of particle physics and is associated with a gauge theory based upon the gauge group structure $SU_3^c \times SU_2 \times U_1$. The essential ingredients of the SM the gauge bosons, photon (γ), weakons ($W^{\pm}, Z^0$), and the eight gluons. Interaction with the Higgs field gives mass to the weakons and to the charged leptons and quarks and thence to the hadrons, baryons and mesons.

■ 5. Limitations of the Standard Model

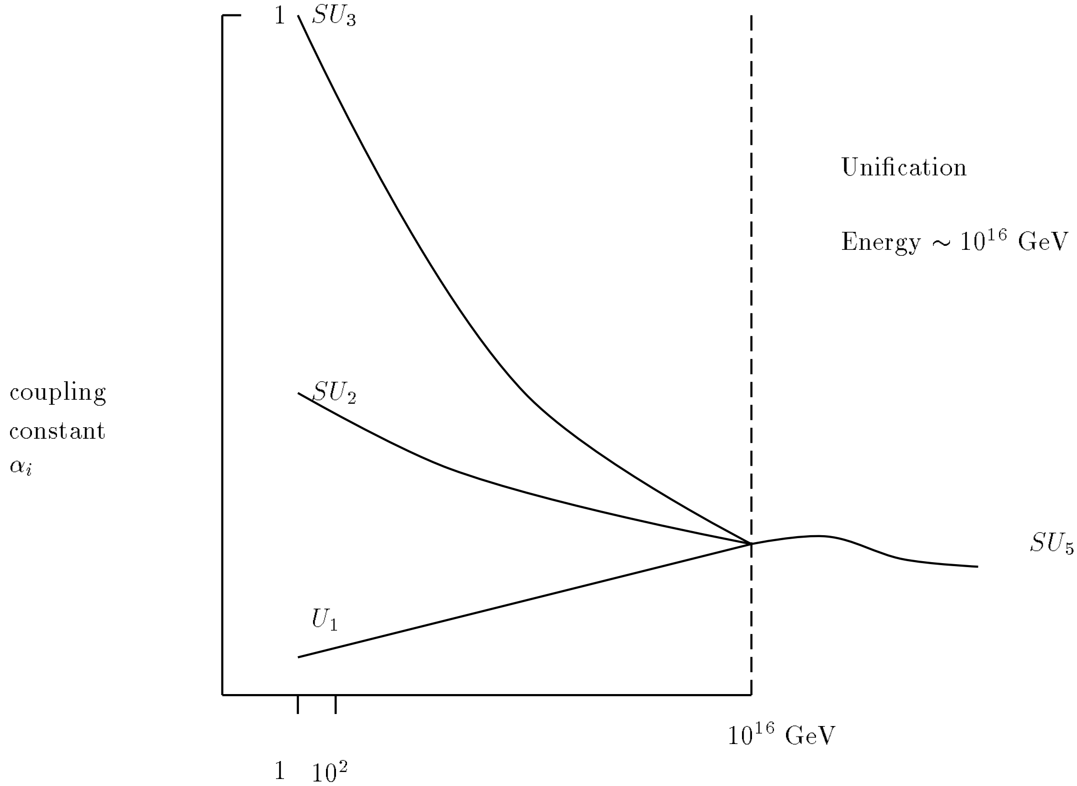
The SM has been extraordinarily successful and has had many predictive successes. Nevertheless, for any theory we can never offer a final and complete proof rather, in the Popperian view, we must attempt to falsify the theory and try to assess its limitations.

The most serious shortcoming is our failure, as yet, to experimentally identify the Higgs boson. This is one of the key objectives of the Large Hadron Collider. The weak interaction angle $sin^2\theta_w$ enters the theory as a parameter. Why does it have the value it has?

There is no reason emergent from the SM as to why there should be three families of quarks and leptons. Charge quantisation, and in particular the relationship of the quark to lepton charges, is outside the theory of the SM. There is no reason given as to why the quarks and leptons have their particular masses and how these are related to those of the $W^{\pm}$, Z^0 bosons. The SM model gives no relationship between the leptons and hadrons or of the relative strength of the electroweak and strong interaction.

■ 6. The first Grand Unified Theory - SU_5

The first attempts to go beyond the SM involved trying to develop a **G**rand **U**nified **T**heory that encompassed both the electroweak and strong interactions into a single theory. To that end one seeks a higher symmetry in which at some high unification energy the coupling constants that measure the strengths of the electromagnetic, weak and strong interactions all tend to a common value. This means that the coupling "constants" are in fact energy dependent. Studies of the coupling constants as a function of energy suggest a unification energy of $\sim 10^{16} GeV$, an energy well beyond any conceivable particle accelerator. Such a unification energy domain could only have been reached in the very early universe (at $t \sim 10^{-36}$ seconds!). In that sense, cosmology and particle physics also become unified.



A pictorial representation of grand unification

Below the grand unification energy the symmetry is broken down to that of the SM. The physics of our epoch comes from the symmetry breaking. The GUT should contain the SM after the symmetry breaking. The first GUT theory was suggested by Georgi and Glashow in 1974 (*Phys. Rev. Lett.* **32**, *438(1974))* Their GUT involved finding the smallest mathematical symmetry group that contained the standard model gauge group $SU_3^c \times SU_2 \times U_1$ as a subgroup. This turned out to be the group SU_5. Their theory had some immediate successes - charge was naturally quantised, as expected, but more importantly the third integral charges of the quarks came out naturally as did their relationship to the charges of the leptons. Furthermore, the right-handed SU_2 doublets of leptons were found to be partnered with a right-handed singlet of quarks. Thus the SU_5 GUT *predicted* that the right-handed quarks are singlets under SU_2. Another attractive feature was that the entire set of 15 states associated with a single quark-lepton family could be accommodated in such a way that previously disturbing anomalies tha plaqued earlier theories automatically cancelled out.

Whereas in the SM $\sin\theta_w$ is a parameter the primitive version of the SU_5 GUT gave a prediction of its value. Initially the predicted value seemed in disagreement with experiment until it was realised that the predicted angle was appropriate to the grand unification value and is changed in the symmetry breaking down to the experimental energies at which $\sin\theta_w$ was measured.

■ 7. The Terrible Lepto-quarks!

Initially the SU_5 GUT looked tremendously encouraging and certainly scored some stunning successes. We have already seen that electromagnetic interactions involved a single boson, γ, the weak interactions a further three bosons, $W^{\pm}$, Z^0, and the strong interactions eight bosons, the gluons. What, if any, additional bosons arise in GUT? Technically the answer has to be found in the adjoint representation of SU_5. We find a total of 24 gauge bosons. Among these, not surprisingly are the friendly bosons just referred to and all with the right quantum numbers and colours. But $(24-12) = 12$. Who are the 12 new bosons? We find they divide into two sets of 6 bosons which carry hypercharge, isospin AND colour quantum numbers. There hypercharge is third integral with one set forming a colour triplet and the other a colour anti-triplet. Thus they appear to share quantum numbers associated with leptons and quarks and yet are bosons. These are the lepto-quarks and indeed must arise in some form in any GUT.

What are the consequences of having lepto-quarks in a GUT? The most serious consequence is that they can violate both baryon number conservation ($\Delta B = 0$) and lepton conservation ($\Delta L = 0$). This means that they could cause a proton to decay e.g. $p \rightarrow e^+ + \pi^0$. Calculations of the lifetime of a proton were made on the basis of the SU_5 GUT giving the upper limit prediction of $\tau_{n,p} \sim 10^{31} years$. This is a tremendously long lifetime even compared to that of the universe. Nevertheless, experiments have shown that the lifetime is certainly longer than $10^{32} years$ and thus the SU_5 GUT theory is falsified.

■ 8. Grander Grand Unified Theories?

The simple SU_5 GUT came very close to providing a GUT and its near failure inspires further attempts to construct other GUTs. Note however, the SU_5 GUT gives no solution to the family problem, no clear prediction for the mass of the Higgs boson and suffers the technical problem of placing the quarks and leptons in two representations of SU_5 whereas one might have hoped for a single representation. This latter problem can be overcome by using groups larger than SU_5 as is indeed the case for SO_{10} models but that is achieved at the expense of the introduction of a right-handed neutrino which has not been observed. In the SM the masses of the neutrinos are precisely zero. Current experiments at Los Alamos suggest the possibility of a small mass ($\sim 2.4eV$) but that is still a very tentative result. Finally, it should be noted that the GUT's do not include the fourth interaction - gravity.

■ 9. Concluding Remarks

The SM has been remarkably successful but is certainly not the final theory. Symmetry has played a key role in the development of the SM but have we exploited all possible symmetries? I shall explore this topic in the next chapter.

■ Seeing Muons!

I suggest you attempt the following experiment and report your results at the next lecture. Muons are produced in the atmosphere from cosmic rays. As they travel through the liquid in your eyeballs they emit a cone of Ćerenkov radiation. When you go to bed at night and the room is dark close your eyes gently and wait sometime, avoiding falling asleep!, and see if you can see an occasional flash of light. Note, this experiment explains the flashes of light seen by astronauts when in darkness.

■ Worth Quoting?

> *"When a thing was new, people said, 'It is not true'. Later, when its truth became obvious, people said, 'Anyhow, it is not important' and when its importance could no longer be denied, people said, 'Anyway, it is not new'".* (William James, philosopher)
>
> *"... highlights the dilemma confronting Australian science as it is dragged away from probing the deep questions to what writer Barry Jones once dismissively termed, 'panel beating for industry'"* (The Australian August 1995)
>
> *"Once a sage was asked why scholars always flock to the doors of the rich, whilst the rich are not inclined to call at the doors of scholars. 'The scholars' he answered, 'are well aware of the use of money, but the rich are ignorant of the nobility of science"* (Al-Biruni, 973-1048)
>
> Perhaps the motto for particle physics should be *"Freedom brings constraints!"*

Chapter TwentyOne

Towards a Theory Of Everything?

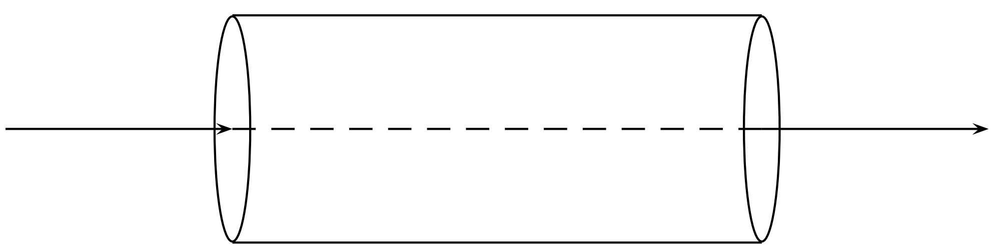
"Anyone who believes that exponential growth can continue indefinitely in a finite world is either a madman or an economist"
Prof. Kenneth Boulding (economist)

SYNOPSIS

In spite of our greatly increased understanding of the structure of matter in terms of the Standard Model a final theory still eludes us though some view the ultimate construction of a **T**heory **Of** **E**verything (**TOE**) as within the forseeable future. In this final chapter we sketch some of the features of current attempts and some of their associated problems and thus bring to an end our never ending story.

■ 1. The Kaluza-Klein Attempt

Maxwell's unification of electricity and magnetism was essentially completed with Einstein's special theory of relativity. Newton had introduced his theory of gravity which in a sense was completed with Einstein's general theory of relativity. At the beginning of this century these two theories encompassed all known forces. Could these two theories be unified into a single coherent theory? Einstein had introduced the concept of four-dimensional spacetime with three spatial and one time variable. The school teacher, Kaluza, wondered if you could consider a five-dimensional spacetime and would it lead to the desired unification. There seemed to be no evidence for such an additional dimension and yet Kaluza realised that he could produce a unified theory of electromagnetism and gravitation. Kaluza viewed the ordinary spacetime dimensions as infinitely extendable whereas his fifth dimension was restricted to a very small range rather like a long pipe, or tube, very long but very narrow width as if the additional dimension is wrapped around itself as illustrated below:



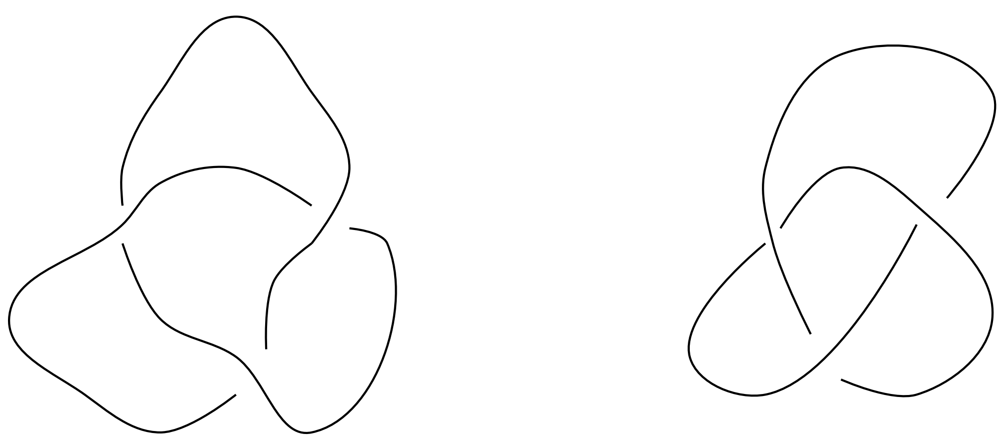
In a sense the familiar four-dimensional spacetime is viewed as a projection from a higher dimensional spacetime. Kaluza wrote up his suggestion and mailed it to Einstein to get his opinion, a year later Einstein wrote to Kaluza saying he was intrigued by the idea and would recommend its publication (T. Kaluza, *Sitzungsber. Preus Akad. Wis. P-M* 966 (1921)). Further development was made by Oscar Klein (O. Klein, Quantentheorie und fünfdimensionale Relativitätstheorie, *Z. Physik* **37**, 895 (1926)) and it became known as the Kaluza-Klein five-dimensional theory. However, thanks to the work of Curie, Rutherford and others it became evident that there were other forces beyond just gravitational and electromagnetic and the Kaluza-Klein theory was largely abandoned.

■ 2. Knots and Strings

Physics seldom develops along predictable directions. Ideas often arise only to be discarded at sometime and to be revived sometimes a century or more later. Often ideas are discarded as been incorrect or irrelevant only to reappear with a new interpretation and to assume a key importance. The theory of strings and knots in physics and mathematics provides an excellent illustration.

In 1867 Lord Kelvin (Also known as W. Thomson) attempted to understand the diverse properties of atoms as vortices in the hypothesised aether (*On vortex atoms*, Phil. Mag. **34**, 15-24 (1867)). Kelvin wondered if the different elements could be interpreted as knots in the vortices of the aether. Could a classification of knots lead in turn to a classification of the atoms of the emergent periodic table? Peter Tait, using pencil, eraser and paper, attempted such a classification exhausting himself at knots with ten crossings. This was to be the start of the mathematical theory of knots. The field progressed slowly over many decades. Were some of Tait's knots that seemed different really different or could two apparently distinct knots simply be transformed into each other without cutting the string making up the knots (technically can one knot be topologically deformed into the other) ? Many mathematicians had spent their careers studying the properties of knots and their classification.

Partially successful attempts were made but even in the early 1980's there were knots in Tait's collection where the question was unresolved. In the 1920's Alexander defined a knot polynomial that distiguish certain pairs of knots. If the knot polynomial was different for two knots then the two knots were distinct, but if the knot polynomial was the same one could not infer that the knots were equivalent. Thus the trefoil knot and its mirror image shown below were clearly distinct knots and yet they possessed the same knot polynomial.



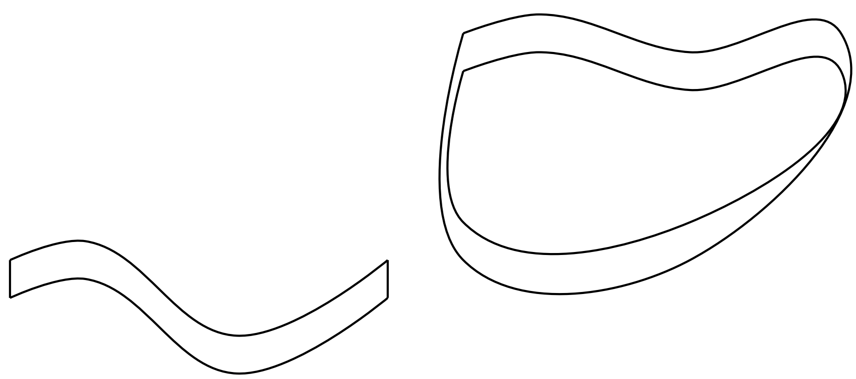
(a) The Trefoil Knot (b) The Mirror Image.

The big breakthrough came in 1984 with the announcement by Vaughan Jones of a new knot polynomial which immediately resolved the undecided knots of Tait's collection including the trefoil knot. Jones was not a knot theorist but was interested in a seemingly unrelated area of mathematics, von Neumann algebras, coming to the subject of knots from a totally unexpected direction. Knots is now a subject of intense study not only by mathematicians but also physicists and indeed biologists. Knots were in a very real sense the forerunners of the modern theory of strings in physics.

■ 3. Strings

Up until the 1970's theories of elementary particles involved the properties of point-like objects with no sensible extension. String theories wre first developed as models of hadrons - quarks and antiquarks linked by a "string", the quarks and antiquarks being the ends of the "string". In such a model the strings can oscillate and the different modes of oscillation or "excitations" were to lead to a spectrum of the hadron resonances. These theories seemed to lead nowhere and were, like Kaluza-Klein, largely abandoned. The subject underwent a significant revival in 1971 with attempts to develop string models, not for describing hadrons, but rather the elementary particles themselves and to give them some spatial extension. Now the "excitations" of the string were to be used to describe the spectrun of the elementary particles themselves.

The relevant particles to be described would be the gauge bosons such as the spin 0 Higgs, the spin 1 gluons and photon, and the spin 2 massless graviton and the fermions - the spin $\frac{1}{2}$ leptons and quarks. The first problem to be solved was "How to you produce a string theory encompassing *both* bosons and fermions?"



■ 4. O'Raifeartaigh says "No go"

In 1965 O'Raifeartaigh examined the possibility of combining the symmetries associated with internal quantum numbers such as spin, hypercharge etc with the symmetries associated with Lorentz invariance required in a relativistically correct theories. He sought a symmetry group that combined the features of the internal symmetries with Lorentz symmetries and produced a famous (or infamous) No-go theorem that said such a task was impossible. One could not combine into a single theory bosons (integer spin) and fermions (half-integer spin).

■ 5. The Open Boson String

The earliest string model involved an open string (i.e. a string with free ends). This model immediately gave rise to a number of unpleasant features. It was constructed as a bosonic string (technically it was constructed from boson creation operators) and hence its excitation spectrum could only create boson states. Worse, its lowest state, or vacuum state was found to have a massed squared that was negative - a so called tachyonic state having negative mass and violating cherished ideas of causality and having particles travelling at speeds necessarily greater than the speed of light. Of course with such a string there was no possibility of generating fermions. Clearly the concept of the string needed to be developed further and given a fermion sector as well as a boson sector.

■ 6. New Strings

1971 saw the first steps in starting to produce more realistic string models. Neveu and Schwarz and Ramond independently constructed two new string models. Each contained both bosonic and fermionic features but were inherently different structures with seemingly no direct connection. The Neveu-Schwarz string still suffered from having a tachyonic ground state. Neither string was able to evade O'Raifeartaigh's no-go theorem and hence could not relate the boson states of the string to its fermion states.

■ 7. Meanwhile back in 1829

Back in 1829 the mathematician C. G. J. Jacobi had noted some very strange identities that he termed *aeqatio identica satis abstrusa* one such being of the form

$$\frac{1}{2}q^{-\frac{1}{2}}\left(\prod_{n=1}^{\infty}(1+q^{n-\frac{1}{2}})^8-\prod_{n=1}^{\infty}(1-q^{n-\frac{1}{2}})^8\right)=8\prod_{n=1}^{\infty}(1+q^n)^8 \qquad (1)$$

The details need not concern us but such an identity appears both surprising and unmotivated. At first sight it appears of no significance and should be left among Jacobi's collected works.

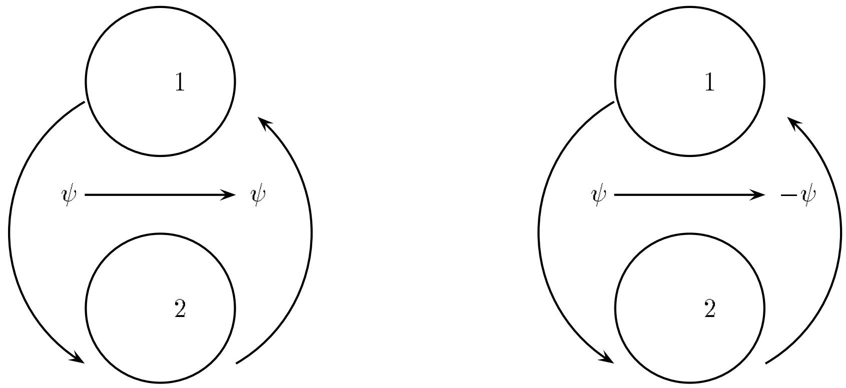
■ 8. Jacobi Returns

Once again string models seemed fraught with difficulties. A hint of a way out came in 1977 from a surprising application of Jacobi's identity which showed that in a ten dimensional spacetime (nine space and one time) it was possible to combine the Neveu-Schwarz and Ramond strings in such a manner that the the number of physical boson states in the boson sector of the Neveu-Schwarz model the fermion Ramond sectors were equal at each mass level. This new string model became known as the Neveu-Schwarz-Ramond string. The important novel features of the Neveu-Schwarz-Ramond string were the existence of an apparent supersymmetry between the boson and fermion sectors, the introduction of higher spacetime dimensions, the freedom from a tachyonic ground state. However, the Neveu-Schwarz-Ramond string still did not circumvent O'Raifeartaigh's no-go theorem. [We (R. J. Farmer, R. C. King and B. G. Wybourne, *Spectrum-generating functions for strings and superstrings*, J. Phys. **A21**, 3979-4007 (1988)) have given a number of examples of the occurrence of the Jacobi identity and indeed other obscure Jacobi identities in connection with strings and superstrings]

■ 9. Exchanging Bosons and Fermions

If two identical bosons are interchanged the sign of the wavefunction is unchanged. The wavefunction is said to be totally symmetric with respect to the interchange of any pair of identical bosons and this is the basis of Bose-Einstein statistics. In is also the reason why lasers are possible and can produce quantum coherent light. (Remember photons have spin 1 and are hence bosons. There is no restriction on the number of bosons that can occupy a given quantum state.)

If two identical fermions are interchanged the sign of the wavefunction is changed. This prevents two fermions being in the same quantum state as in the Pauli exclusion principle. The wavefunction must be totally antisymmetric with respect to the interchange of any pair of fermions.



(a) Bose-Einstein statistics (b) Fermi-Dirac staistics.

■ 10. Supersymmetry

O'Raifeartaigh's no-go theorem had a loophole in it. It excluded the possibility of symmetry transformations that turned a boson into a fermion and vice versa. So-called supersymmetry transformations. Such a supersymmetry transformation then relates bosons to fermions.

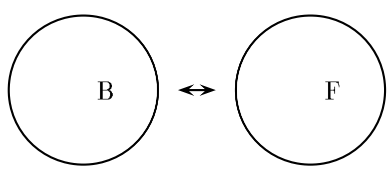


Illustration of the supersymmetry transformation.

■ 11. Awful Infinities

Both QED and QCD suffer from the appearance of highly divergent terms that lead to awful infinities. These arise when one attempts to include quantum corrections for a given process associated with virtual particles, discussed earlier. For example if one attempts to calculate the mass of a Higgs boson one must include the effects of virtual particles such as quark-antiquark fermionic pairs and $W^+ - W^-$ bosonic pairs as shown below:-

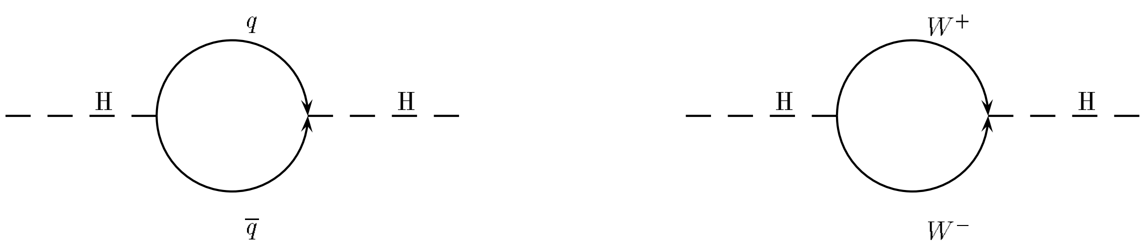


Fig. (1) (a) Virtual quark-antiquark pair (b) Virtual $W^+ - W^-$ pair

Techniques have been developed (so-called *renormalisation techniques*) that still allow one to extract precise results for such things as energy levels, decay rates etc, often in quite remarkable agreement with experiment. Nevertheless, such jugglings with infinities creates a feeling of unease.

■ 12 Supersymmetry to the Rescue

We have seen that fermions and bosons have different statistics leading to the two contributions in Fig. 1 having the opposite sign. Thus we can expect some cancellation when we add the two effects but how complete is the cancellation? Supersymmetry theories relate bosons and fermions and hence the degree of cancellation can be made precise. Indeed in some supersymmetric theories the cancellation is exact and there are no awful infinities left. One of the striking successes of supersymmetric theories has been the prediction of the weak interaction angle $\sin^2\theta_W$ with almost unbelievable precision, certainly far better than any non-supersymmetric theory.

■ 13. Enter Supergravity

Soon after the introduction of supersymmetry transformations it was realised that there was a real possibility of producing a theory of gravity. The big stir in the 1970's following upon the introduction of supersymmetry was the realisation that repeated applications of the supersymmetry operations result in the translation of particles (more precisely Poincaré transformations) and hence such supersymmetric theories, that became known as *supergravity theories*, automatically built in the effects of gravity. The gauge field associated with gravity is associated with the *graviton*, a massless spin 2 particle, as yet unobserved. (direct detection of gravitons is most unlikely but they do carry with them the possibly experimentally verifiable *gravitational waves* which will be the focus of a number of experiments in early next century. Local supersymmetry contains the graviton among its gauge fields and thus includes Einstein's theory of gravity.

Supergravity avoids O'Raifeartaigh's no-go theorem and hence can have connections between states of different spin and thus multiplets containing both bosons and fermions. Such theories are more general than GUT theories that attempt to unify QCD with electroweak theory. GUT theories involve unification of electromagnetic, weak and strong interactions at an energy of $\sim 10^{15} GeV/c^2$. What is the scale at which unification occurs with the gravitational coupling becoming equivalent to that of the other three forces?

■ 14. The Planck Numbers

The full properties of strings take place at the so-called Planck scale which involve special quantites derived from the three fundamental constants:-

$$\begin{aligned} \text{Planck's constant} \quad & h = 6.626 \times 10^{-34} Js \\ \text{Newton's constant} \quad & G = 6.672 \times 10^{-11} m^3 kg^{-1} s^{-2} \\ \text{Speed of light} \quad & c = 2.9979 \times 10^8 ms^{-1} \end{aligned}$$

From these three constants we can construct three Planck numbers:-

$$\begin{aligned} \text{Planck mass} \quad & M_P = \sqrt{\frac{h}{Gc^3}} kg \sim 10^{19} GeV/c^2 \\ \text{Planck length} \quad & M_L = \sqrt{\frac{Gh}{c^3}} \sim 10^{-35} m \\ \text{Planck time} \quad & T_P = \sqrt{\frac{Gh}{c^5}} \sim 10^{-43} s \end{aligned}$$

It is believed that the ultimate unification occurs at an energy equivalent to that of the Planck mass ($10^{19} GeV$) some four orders of magnitude above the energy involved in GUT.

■ 15. Supersymmetric Partners

Supersymmetric theories have many desirable properties. Many of the horrible infinities of non-supersymmetric theories disappear in supersymmetric theories. Part of this desirable feature comes about from the requirement of supersymmetric theories that bosons and fermions be linked and to do this it is necessary that for every particle there exists a supersymmetric particle with the same set of quantum numbers but differing in spin by a half-integer. Thus the spin 2 graviton is accompanied by a spin $\frac{3}{2}$ gravitino, the spin 1 photon by a spin $\frac{1}{2}$ photino etc. These supersymmetric partners are expected to be much more massive than their non-supersymmetric partner.To date there is no direct evidence for their existence but supersymmetric theories appear to be in better accord with experiments than the usual GUT's. The detection of the supersymmetric partners, along with the Higgs bosons, is a primary objective of the European Large Hadron Collider and of the planned upgrading of the European Large Electron-Positron collider (LEP2). For the latter it could be 1996 and for the former 2003. Of one thing I am sure is that physics of the next century will be just, if not more, as exciting as physics has been this century.

■ 16. The M-theory Revolution of 1995

1. The entire situation changed in mid-1995 heralding the second string revolution. I sketch only the briefest of details. This involved the discovery of new symmetries associated with superstring theories. Already startling results have appeared - in some cases calculations, previously beyond any supercomputer, have been reduced to pencil and paper calculations. All the various string theories turn out to be related and hence cannot be regarded as distinct theories but rather one theory becomes the limiting form of one of the other theories. The key idea is known as "*String Duality.*
2. Recall Maxwell's equations of electromagnetism, in the absence of sources, (i.e. currents and charges)

$$\begin{aligned} \nabla \cdot \mathbf{E} = 0 \quad & \nabla \times \mathbf{E} + \dot{\mathbf{B}} = 0 \\ \nabla \cdot \mathbf{B} = 0 \quad & \nabla \times \mathbf{B} - \dot{\mathbf{E}} = \mathbf{0} \end{aligned}$$

   The equations are invariant under $\mathbf{E} \rightarrow \mathbf{B}, \quad \mathbf{B} \rightarrow -\mathbf{E}$ which exchanges electric and magnetic fields. This is an example of *duality symmetry.* If charged particles are added to the equations, the duality symmetry is only preserved only if *both* electric charges *and* magnetic monopoles.
3. The possible existence of magnetic momopoles was considered by Dirac leading to the quantization condition which relates the electric charge e to the magnetic charge g as, with $\hbar = 1$,

$$eg = 2\pi n \qquad n = \pm 1.$$

In that case duality exchanges not only the electric and magnetic fields but also electric and magnetic charges. Since eg is fixed and $e \ll 1$ while $g \gg 1$ we can regard electrodynamics in terms of electric charges as a *weakly coupled* theory while if it was based upon magnetic charges it would be a *strongly coupled* theory and would require, unlike conventional QED, a very complicated and difficult non-perturbative treatment.

4. The M-theory revolution of 1995 is the recognition of the duality symmetry as a symmetry of string theory. The key idea is that the strongly coupled limit of any string theory is equivalent to the weakly coupled limit of some other string theory. Thus all string theories become connected and all are subsumed in an eleven-dimensional M-theory with the duality symmetry manifest. To atomic physicists the analogue can be seen in $LS-$coupling and $jj-$coupling. In $LS-$coupling the calculation of Coulomb matrix elements is "easy" while the calculation of spin-orbit interactions is "hard". Conversely in $jj-$coupling the calculation of Coulomb matrix elements is "hard" while the calculation of spin-orbit interactions is "easy".
5. The M-theory is developing very rapidly. One of the first applications has been in the application of quantum theory to black holes. Over the past 20 years it has been thought that one could not successfully combine quantum theory and general relativity in nthe description of black holes and hence some modification of quantum theory would be required. Calculations in the past few months, based upon the concept of duality and strings has shown that one can indeed give a consistent treatment of the quantum theoretical description of black holes without the need to modify the basic ideas of quantum theory.

■ 17. Ultimate Symmetries

The developments of M-theory are startling and may indicate that we are on the path to the discovery of the ultimate symmetries of the laws of the universe. There is still a long way to go and history shows that it is always dangerous to assume we are reaching the end of the road. With the exciting experiments planned for the next century I am confident that it will be possible to make tremendous progress in understanding very basic properties of the universe, its past, present and future. Much imaginative and daring thought will be required with ultimate constraints coming from experiment, though significant areas will remain unverifiable as the energies that occurred in the very early universe will be forever beyond human possibilities. Poets, musicians, creators of great literature will all be required to express our story for our story is a never ending story. Finis...

> ... *If you look at the history of 20th century physics, you will find that the symmetry concept as a most fundamental theme, occupying center stage in today's theoretical physics. We cannot tell what the 21st century will bring us but I feel safe to say that for the next twenty years many theoretical physicists will continue to try variations on the fundamental theme of symmetry at the very foundations of our theoretical understanding of the structure of the physical universe*
>
> – C. N. Yang *Chinese J. Phys.* **32**, 1437 (1994)

The only questions worth asking are the unanswerable ones
– John Ciardi *Saturday Review-World (1973)*

For every complex question there is a simple answer — and it's wrong.
– H. L. Mencken

Haere koe i te ara a taihoa, ki a tae ai koe ki aua atu.
Travel on the pathway of by and by, so that you may reach goodness knows where.
– Maori proverb