Tables of the inverses of certain series

Introduction

We consider the four infinite series of $q-$dependent series defined by

$$A(q) = \sum_{\alpha} (-q)^{\omega_\alpha} [\alpha]_{O(N)} \ or \ < \alpha >_{Sp(N)} \tag{1}$$

$$C(q) = \sum_{\gamma} (-q)^{\omega_\gamma} [\gamma]_{O(N)} \ or \ < \gamma >_{Sp(N)} \tag{2}$$

$$E(q) = \sum_{\varepsilon} (-q)^{\frac{\omega_\varepsilon + r_\varepsilon}{2}} [\varepsilon]_{O(N)} \ or \ < \varepsilon >_{Sp(N)} \tag{3}$$

$$H(q) = \sum_{\zeta} (-q)^{\omega_\zeta} \{\tilde{\zeta}; \zeta\}_{U(N)} \tag{4}$$

where in Frobenius notation

$$(\alpha) = \begin{pmatrix} a_1 & a_2 & \ldots & a_r \\ a_1 + 1 & a_2 + 1 & \ldots & a_r + 1 \end{pmatrix}, \quad (\gamma) = \begin{pmatrix} c_1 + 1 & c_2 + 1 & \ldots & c_r + 1 \\ c_1 & c_2 & \ldots & c_r \end{pmatrix},$$

(ε) is any self-conjugate partition and (ζ) is any partition. For any partition (λ) r is its *rank* and ω_λ its *weight*. Terms encased in square brackets [,] are interpreted as characters of the full orthogonal group $O(N)$, those encased in angular brackets $<,>$ for the symplectic group $Sp(N)$ and those encased in curly brackets $\{,\}$ for the unitary group $U(N)$.

The terms in the series are determined up to a cutoff power of q and likewise for their inverses. In practice N is chosen to be sufficiently large so that no application of modification rules is required.

Method of computation

The tables were produced using the newest version of SCHUR which is able to generate the terms in the series and compute their inverses. Each series was evaluated using a function such as below (for the $H(q)$ series)

```
sb_rev true
sb_tex true
columns4
supoutpt false
log'mac7.tex'
gr2u1u12
mac h,t,f,f,f,t,7
return
return
inv7last
log''
sb_rev false
sb_tex false
set_pwt100
```

stop The first three statements produce the output in Plain TEX, written to the logfile 'mac7.tex', in a form from which tables may be readily constructed. The group was set as $U(1) \times U(12)$ with the exponents of q being given as the integers in the $U(1)$ irreps. The instruction 'mac h,t,f,f,f,t,7' determines the terms in $H(q)$ up to q^7, 'return' inserts a blank line, while 'inv7last' computes the inverse, $H(q)^{-1}$, up to q^7 finally we restore some default values. 'mac7.tex' is then edited with a texteditor to find and replace each $\{n\}$ of $U(1)$ by q^n.

The most time consuming, a memory consuming, computation was that for $H(q)^{-1}$. Terms to q^6 took 82minutes on a SUN Sparc 5 and 15minutes on a SUN Ultra 1. While terms to q^7 were calculated on the latter machine in a time of ~ 8 hours (I am grateful to S. Christensen for performing this calculation).

The Tables

$$A(q) =$$

$< 0 >$	$- q < 1^2 >$	$+ q^2 < 21^2 >$	$- q^3 < 2^3 >$
$- q^3 < 31^3 >$	$+ q^4 < 32^2 1 >$	$+ q^4 < 41^4 >$	$- q^5 < 3^2 2^2 >$
$- q^5 < 42^2 1^2 >$	$- q^5 < 51^5 >$	$+ q^6 < 3^4 >$	$+ q^6 < 432^2 1 >$
$+ q^6 < 52^2 1^3 >$	$+ q^6 < 61^6 >$		

$$A(q)^{-1} =$$

$< 0 >$	$+ q < 1^2 >$	$+ q^2 < 0 >$	$+ q^2 < 1^2 >$
$+ q^2 < 1^4 >$	$+ q^2 < 2 >$	$+ q^2 < 2^2 >$	$+ q^3 < 0 >$
$+ 4q^3 < 1^2 >$	$+ q^3 < 1^4 >$	$+ q^3 < 1^6 >$	$+ q^3 < 2 >$
$+ 2q^3 < 21^2 >$	$+ q^3 < 2^2 >$	$+ q^3 < 2^2 1^2 >$	$+ q^3 < 31 >$
$+ q^3 < 3^2 >$	$+ 4q^4 < 0 >$	$+ 6q^4 < 1^2 >$	$+ 5q^4 < 1^4 >$
$+ q^4 < 1^6 >$	$+ q^4 < 1^8 >$	$+ 5q^4 < 2 >$	$+ 5q^4 < 21^2 >$
$+ 2q^4 < 21^4 >$	$+ 6q^4 < 2^2 >$	$+ 2q^4 < 2^2 1^2 >$	$+ q^4 < 2^2 1^4 >$
$+ q^4 < 2^3 >$	$+ q^4 < 2^4 >$	$+ 2q^4 < 31 >$	$+ q^4 < 31^3 >$
$+ 2q^4 < 321 >$	$+ q^4 < 3^2 >$	$+ q^4 < 3^2 1^2 >$	$+ q^4 < 4 >$
$+ q^4 < 42 >$	$+ q^4 < 4^2 >$	$+ 4q^5 < 0 >$	$+ 18q^5 < 1^2 >$
$+ 9q^5 < 1^4 >$	$+ 5q^5 < 1^6 >$	$+ q^5 < 1^8 >$	$+ q^5 < 1^{10} >$
$+ 8q^5 < 2 >$	$+ 17q^5 < 21^2 >$	$+ 6q^5 < 21^4 >$	$+ 2q^5 < 21^6 >$
$+ 10q^5 < 2^2 >$	$+ 11q^5 < 2^2 1^2 >$	$+ 2q^5 < 2^2 1^4 >$	$+ q^5 < 2^2 1^6 >$
$+ 2q^5 < 2^3 >$	$+ 2q^5 < 2^3 1^2 >$	$+ q^5 < 2^4 >$	$+ q^5 < 2^4 1^2 >$
$+ 10q^5 < 31 >$	$+ 3q^5 < 31^3 >$	$+ q^5 < 31^5 >$	$+ 8q^5 < 321 >$
$+ 2q^5 < 321^3 >$	$+ 2q^5 < 32^2 1 >$	$+ 6q^5 < 3^2 >$	$+ 2q^5 < 3^2 1^2 >$
$+ q^5 < 3^2 1^4 >$	$+ q^5 < 3^2 2 >$	$+ q^5 < 3^2 2^2 >$	$+ q^5 < 4 >$
$+ 2q^5 < 41^2 >$	$+ 2q^5 < 42 >$	$+ q^5 < 421^2 >$	$+ 2q^5 < 431 >$
$+ q^5 < 4^2 >$	$+ q^5 < 4^2 1^2 >$	$+ q^5 < 51 >$	$+ q^5 < 53 >$
$+ q^5 < 5^2 >$	$+ 13q^6 < 0 >$	$+ 31q^6 < 1^2 >$	$+ 29q^6 < 1^4 >$
$+ 10q^6 < 1^6 >$	$+ 5q^6 < 1^8 >$	$+ q^6 < 1^{10} >$	$+ q^6 < 1^{12} >$
$+ 25q^6 < 2 >$	$+ 41q^6 < 21^2 >$	$+ 21q^6 < 21^4 >$	$+ 6q^6 < 21^6 >$
$+ 2q^6 < 21^8 >$	$+ 35q^6 < 2^2 >$	$+ 27q^6 < 2^2 1^2 >$	$+ 12q^6 < 2^2 1^4 >$
$+ 2q^6 < 2^2 1^6 >$	$+ q^6 < 2^2 1^8 >$	$+ 14q^6 < 2^3 >$	$+ 7q^6 < 2^3 1^2 >$
$+ 2q^6 < 2^3 1^4 >$	$+ 7q^6 < 2^4 >$	$+ 2q^6 < 2^4 1^2 >$	$+ q^6 < 2^4 1^4 >$
$+ q^6 < 2^5 >$	$+ q^6 < 2^6 >$	$+ 24q^6 < 31 >$	$+ 18q^6 < 31^3 >$
$+ 3q^6 < 31^5 >$	$+ q^6 < 31^7 >$	$+ 28q^6 < 321 >$	$+ 10q^6 < 321^3 >$
$+ 2q^6 < 321^5 >$	$+ 9q^6 < 32^2 1 >$	$+ 2q^6 < 32^2 1^3 >$	$+ 2q^6 < 32^3 1 >$
$+ 12q^6 < 3^2 >$	$+ 13q^6 < 3^2 1^2 >$	$+ 2q^6 < 3^2 1^4 >$	$+ q^6 < 3^2 1^6 >$
$+ 3q^6 < 3^2 2 >$	$+ 2q^6 < 3^2 21^2 >$	$+ 2q^6 < 3^2 2^2 >$	$+ q^6 < 3^2 2^2 1^2 >$
$+ q^6 < 3^3 1 >$	$+ q^6 < 3^4 >$	$+ 6q^6 < 4 >$	$+ 7q^6 < 41^2 >$
$+ 2q^6 < 41^4 >$	$+ 12q^6 < 42 >$	$+ 4q^6 < 421^2 >$	$+ q^6 < 421^4 >$
$+ 4q^6 < 42^2 >$	$+ q^6 < 42^3 >$	$+ 8q^6 < 431 >$	$+ 2q^6 < 431^3 >$
$+ 2q^6 < 4321 >$	$+ 6q^6 < 4^2 >$	$+ 2q^6 < 4^2 1^2 >$	$+ q^6 < 4^2 1^4 >$
$+ q^6 < 4^2 2 >$	$+ q^6 < 4^2 2^2 >$	$+ 2q^6 < 51 >$	$+ q^6 < 51^3 >$
$+ 2q^6 < 521 >$	$+ 2q^6 < 53 >$	$+ q^6 < 531^2 >$	$+ 2q^6 < 541 >$
$+ q^6 < 5^2 >$	$+ q^6 < 5^2 1^2 >$	$+ q^6 < 6 >$	$+ q^6 < 62 >$
$+ q^6 < 64 >$	$+ q^6 < 6^2 >$		

$$C(q) =$$

$$
\begin{aligned}
&< 0 > &&- q < 2 > &&+ q^2 < 31 > &&- q^3 < 3^2 > \\
&- q^3 < 41^2 > &&+ q^4 < 431 > &&+ q^4 < 51^3 > &&- q^5 < 4^2 2 > \\
&- q^5 < 531^2 > &&- q^5 < 61^4 > &&+ q^6 < 4^3 > &&+ q^6 < 5421 > \\
&+ q^6 < 631^3 > &&+ q^6 < 71^5 >
\end{aligned}
$$

$$C(q)^{-1} =$$

$$
\begin{aligned}
&< 0 > &&+ q < 2 > &&+ q^2 < 0 > &&+ q^2 < 1^2 > \\
&+ q^2 < 2 > &&+ q^2 < 2^2 > &&+ q^2 < 4 > &&+ q^3 < 0 > \\
&+ q^3 < 1^2 > &&+ 4q^3 < 2 > &&+ q^3 < 21^2 > &&+ q^3 < 2^2 > \\
&+ q^3 < 2^3 > &&+ 2q^3 < 31 > &&+ q^3 < 4 > &&+ q^3 < 42 > \\
&+ q^3 < 6 > &&+ 4q^4 < 0 > &&+ 5q^4 < 1^2 > &&+ q^4 < 1^4 > \\
&+ 6q^4 < 2 > &&+ 2q^4 < 21^2 > &&+ 6q^4 < 2^2 > &&+ q^4 < 2^2 1^2 > \\
&+ q^4 < 2^3 > &&+ q^4 < 2^4 > &&+ 5q^4 < 31 > &&+ 2q^4 < 321 > \\
&+ q^4 < 3^2 > &&+ 5q^4 < 4 > &&+ q^4 < 41^2 > &&+ 2q^4 < 42 > \\
&+ q^4 < 42^2 > &&+ q^4 < 4^2 > &&+ 2q^4 < 51 > &&+ q^4 < 6 > \\
&+ q^4 < 62 > &&+ q^4 < 8 > &&+ 4q^5 < 0 > &&+ 8q^5 < 1^2 > \\
&+ q^5 < 1^4 > &&+ 18q^5 < 2 > &&+ 10q^5 < 21^2 > &&+ q^5 < 21^4 > \\
&+ 10q^5 < 2^2 > &&+ 2q^5 < 2^2 1^2 > &&+ 6q^5 < 2^3 > &&+ q^5 < 2^3 1^2 > \\
&+ q^5 < 2^4 > &&+ q^5 < 2^5 > &&+ 17q^5 < 31 > &&+ 2q^5 < 31^3 > \\
&+ 8q^5 < 321 > &&+ 2q^5 < 32^2 1 > &&+ 2q^5 < 3^2 > &&+ q^5 < 3^2 2 > \\
&+ 9q^5 < 4 > &&+ 3q^5 < 41^2 > &&+ 11q^5 < 42 > &&+ q^5 < 421^2 > \\
&+ 2q^5 < 42^2 > &&+ q^5 < 42^3 > &&+ 2q^5 < 431 > &&+ q^5 < 4^2 > \\
&+ q^5 < 4^2 2 > &&+ 6q^5 < 51 > &&+ 2q^5 < 521 > &&+ 2q^5 < 53 > \\
&+ 5q^5 < 6 > &&+ q^5 < 61^2 > &&+ 2q^5 < 62 > &&+ q^5 < 62^2 > \\
&+ q^5 < 64 > &&+ 2q^5 < 71 > &&+ q^5 < 8 > &&+ q^5 < 82 > \\
&+ q^5 < 10 \ > &&+ 13q^6 < 0 > &&+ 25q^6 < 1^2 > &&+ 6q^6 < 1^4 > \\
&+ q^6 < 1^6 > &&+ 31q^6 < 2 > &&+ 24q^6 < 21^2 > &&+ 2q^6 < 21^4 > \\
&+ 35q^6 < 2^2 > &&+ 12q^6 < 2^2 1^2 > &&+ q^6 < 2^2 1^4 > &&+ 12q^6 < 2^3 > \\
&+ 2q^6 < 2^3 1^2 > &&+ 6q^6 < 2^4 > &&+ q^6 < 2^4 1^2 > &&+ q^6 < 2^5 > \\
&+ q^6 < 2^6 > &&+ 41q^6 < 31 > &&+ 7q^6 < 31^3 > &&+ 28q^6 < 321 > \\
&+ 2q^6 < 321^3 > &&+ 8q^6 < 32^2 1 > &&+ 2q^6 < 32^3 1 > &&+ 14q^6 < 3^2 > \\
&+ 4q^6 < 3^2 1^2 > &&+ 3q^6 < 3^2 2 > &&+ q^6 < 3^2 2^2 > &&+ 29q^6 < 4 > \\
&+ 18q^6 < 41^2 > &&+ q^6 < 41^4 > &&+ 27q^6 < 42 > &&+ 4q^6 < 421^2 > \\
&+ 13q^6 < 42^2 > &&+ q^6 < 42^2 1^2 > &&+ 2q^6 < 42^3 > &&+ q^6 < 42^4 > \\
&+ 9q^6 < 431 > &&+ 2q^6 < 4321 > &&+ q^6 < 43^2 > &&+ 7q^6 < 4^2 > \\
&+ q^6 < 4^2 1^2 > &&+ 2q^6 < 4^2 2 > &&+ q^6 < 4^2 2^2 > &&+ q^6 < 4^3 > \\
&+ 21q^6 < 51 > &&+ 2q^6 < 51^3 > &&+ 10q^6 < 521 > &&+ 2q^6 < 52^2 1 > \\
&+ 7q^6 < 53 > &&+ 2q^6 < 532 > &&+ 2q^6 < 541 > &&+ q^6 < 5^2 > \\
&+ 10q^6 < 6 > &&+ 3q^6 < 61^2 > &&+ 12q^6 < 62 > &&+ q^6 < 621^2 > \\
&+ 2q^6 < 62^2 > &&+ q^6 < 62^3 > &&+ 2q^6 < 631 > &&+ 2q^6 < 64 > \\
&+ q^6 < 642 > &&+ q^6 < 6^2 > &&+ 6q^6 < 71 > &&+ 2q^6 < 721 > \\
&+ 2q^6 < 73 > &&+ 5q^6 < 8 > &&+ q^6 < 81^2 > &&+ 2q^6 < 82 > \\
&+ q^6 < 82^2 > &&+ q^6 < 84 > &&+ 2q^6 < 91 > &&+ q^6 < 10 \ > \\
&+ q^6 < 10\ 2 > &&+ q^6 < 12 \ >
\end{aligned}
$$

$$E(q) =$$

$< 0 >$	$- q < 1 >$	$+ q^2 < 21 >$	$- q^3 < 2^2 >$
$- q^3 < 31^2 >$	$+ q^4 < 321 >$	$+ q^4 < 41^3 >$	$- q^5 < 3^2 2 >$
$- q^5 < 421^2 >$	$- q^5 < 51^4 >$	$+ q^6 < 3^3 >$	$+ q^6 < 4321 >$
$+ q^6 < 521^3 >$	$+ q^6 < 61^5 >$		

$$E(q)^{-1} =$$

$< 0 >$	$+ q < 1 >$	$+ q^2 < 0 >$	$+ q^2 < 1^2 >$
$+ q^2 < 2 >$	$- q^2 < 21 >$	$+ 3q^3 < 1 >$	$- 2q^3 < 1^2 >$
$+ q^3 < 1^3 >$	$- 2q^3 < 2 >$	$+ 2q^3 < 21 >$	$- 2q^3 < 21^2 >$
$- q^3 < 2^2 >$	$+ q^3 < 3 >$	$- 2q^3 < 31 >$	$+ q^3 < 31^2 >$
$+ 4q^4 < 0 >$	$- 6q^4 < 1 >$	$+ 8q^4 < 1^2 >$	$- 6q^4 < 1^3 >$
$+ 2q^4 < 1^4 >$	$+ 8q^4 < 2 >$	$- 13q^4 < 21 >$	$+ 8q^4 < 21^2 >$
$- 3q^4 < 21^3 >$	$+ 4q^4 < 2^2 >$	$- 4q^4 < 2^2 1 >$	$+ q^4 < 2^2 1^2 >$
$+ q^4 < 2^3 >$	$- 6q^4 < 3 >$	$+ 8q^4 < 31 >$	$- 6q^4 < 31^2 >$
$+ 3q^4 < 31^3 >$	$- 4q^4 < 32 >$	$+ 3q^4 < 321 >$	$+ q^4 < 3^2 >$
$+ 2q^4 < 4 >$	$- 3q^4 < 41 >$	$+ 3q^4 < 41^2 >$	$- q^4 < 41^3 >$
$+ q^4 < 42 >$	$- 8q^5 < 0 >$	$+ 28q^5 < 1 >$	$- 35q^5 < 1^2 >$
$+ 29q^5 < 1^3 >$	$- 14q^5 < 1^4 >$	$+ 4q^5 < 1^5 >$	$- 35q^5 < 2 >$
$+ 58q^5 < 21 >$	$- 48q^5 < 21^2 >$	$+ 26q^5 < 21^3 >$	$- 6q^5 < 21^4 >$
$- 31q^5 < 2^2 >$	$+ 32q^5 < 2^2 1 >$	$- 13q^5 < 2^2 1^2 >$	$+ 3q^5 < 2^2 1^3 >$
$- 7q^5 < 2^3 >$	$+ 4q^5 < 2^3 1 >$	$+ 29q^5 < 3 >$	$- 48q^5 < 31 >$
$+ 45q^5 < 31^2 >$	$- 20q^5 < 31^3 >$	$+ 6q^5 < 31^4 >$	$+ 32q^5 < 32 >$
$- 26q^5 < 321 >$	$+ 14q^5 < 321^2 >$	$- 2q^5 < 321^3 >$	$+ 8q^5 < 32^2 >$
$- 2q^5 < 32^2 1 >$	$- 7q^5 < 3^2 >$	$+ 8q^5 < 3^2 1 >$	$- 2q^5 < 3^2 1^2 >$
$- q^5 < 3^2 2 >$	$- 14q^5 < 4 >$	$+ 26q^5 < 41 >$	$- 20q^5 < 41^2 >$
$+ 12q^5 < 41^3 >$	$- 4q^5 < 41^4 >$	$- 13q^5 < 42 >$	$+ 14q^5 < 421 >$
$- 5q^5 < 421^2 >$	$- 2q^5 < 42^2 >$	$+ 4q^5 < 43 >$	$- 2q^5 < 431 >$
$+ 4q^5 < 5 >$	$- 6q^5 < 51 >$	$+ 6q^5 < 51^2 >$	$- 4q^5 < 51^3 >$
$+ q^5 < 51^4 >$	$+ 3q^5 < 52 >$	$- 2q^5 < 521 >$	$+ 41q^6 < 0 >$
$- 116q^6 < 1 >$	$+ 174q^6 < 1^2 >$	$- 154q^6 < 1^3 >$	$+ 96q^6 < 1^4 >$
$- 38q^6 < 1^5 >$	$+ 8q^6 < 1^6 >$	$+ 174q^6 < 2 >$	$- 314q^6 < 21 >$
$+ 304q^6 < 21^2 >$	$- 183q^6 < 21^3 >$	$+ 70q^6 < 21^4 >$	$- 14q^6 < 21^5 >$
$+ 198q^6 < 2^2 >$	$- 226q^6 < 2^2 1 >$	$+ 137q^6 < 2^2 1^2 >$	$- 48q^6 < 2^2 1^3 >$
$- q^6 < 2^3 1^3 >$	$+ 8q^6 < 2^4 >$	$- 2q^6 < 2^4 1 >$	$- 154q^6 < 3 >$
$+ 304q^6 < 31 >$	$- 290q^6 < 31^2 >$	$+ 177q^6 < 31^3 >$	$- 70q^6 < 31^4 >$
$+ 13q^6 < 31^5 >$	$- 226q^6 < 32 >$	$+ 261q^6 < 321 >$	$- 156q^6 < 321^2 >$
$+ 47q^6 < 321^3 >$	$- 8q^6 < 321^4 >$	$- 86q^6 < 32^2 >$	$+ 49q^6 < 32^2 1 >$
$- 14q^6 < 32^2 1^2 >$	$- 8q^6 < 32^3 >$	$+ 75q^6 < 3^2 >$	$- 86q^6 < 3^2 1 >$
$+ 43q^6 < 3^2 1^2 >$	$- 14q^6 < 3^2 1^3 >$	$+ q^6 < 3^2 1^4 >$	$+ 28q^6 < 3^2 2 >$
$- 18q^6 < 3^2 21 >$	$+ q^6 < 3^2 21^2 >$	$+ q^6 < 3^2 2^2 >$	$- 3q^6 < 3^3 >$

$$E(q)^{-1}(Contd)$$

$+\ 96q^6 < 4 >$	$-\ 183q^6 < 41 >$	$+\ 177q^6 < 41^2 >$	$-\ 113q^6 < 41^3 >$
$+\ 41q^6 < 41^4 >$	$-\ 10q^6 < 41^5 >$	$+\ 137q^6 < 42 >$	$-\ 156q^6 < 421 >$
$+\ 80q^6 < 421^2 >$	$-\ 30q^6 < 421^3 >$	$+\ 3q^6 < 421^4 >$	$+\ 43q^6 < 42^2 >$
$-\ 28q^6 < 42^21 >$	$+\ 3q^6 < 42^21^2 >$	$+\ q^6 < 42^3 >$	$-\ 50q^6 < 43 >$
$+\ 49q^6 < 431 >$	$-\ 28q^6 < 431^2 >$	$+\ 4q^6 < 431^3 >$	$-\ 18q^6 < 432 >$
$+\ 3q^6 < 4321 >$	$+\ 8q^6 < 4^2 >$	$-\ 8q^6 < 4^21 >$	$+\ q^6 < 4^21^2 >$
$+\ q^6 < 4^22 >$	$-\ 38q^6 < 5 >$	$+\ 70q^6 < 51 >$	$-\ 70q^6 < 51^2 >$
$+\ 41q^6 < 51^3 >$	$-\ 20q^6 < 51^4 >$	$+\ 5q^6 < 51^5 >$	$-\ 48q^6 < 52 >$
$+\ 47q^6 < 521 >$	$-\ 30q^6 < 521^2 >$	$+\ 7q^6 < 521^3 >$	$-\ 14q^6 < 52^2 >$
$+\ 4q^6 < 52^21 >$	$+\ 13q^6 < 53 >$	$-\ 14q^6 < 531 >$	$+\ 3q^6 < 531^2 >$
$+\ q^6 < 532 >$	$-\ 2q^6 < 54 >$	$+\ 8q^6 < 6 >$	$-\ 14q^6 < 61 >$
$+\ 13q^6 < 61^2 >$	$-\ 10q^6 < 61^3 >$	$+\ 5q^6 < 61^4 >$	$-\ q^6 < 61^5 >$
$+\ 7q^6 < 62 >$	$-\ 8q^6 < 621 >$	$+\ 3q^6 < 621^2 >$	$+\ q^6 < 62^2 >$
$-\ q^6 < 63 >$			

$$H(q) =$$

$$
\begin{aligned}
&\{0;0\} & -\,& q\{1;1\} & +\,& q^2\{2;1^2\} & +\,& q^2\{1^2;2\} \\
-\,& q^3\{3;1^3\} & -\,& q^3\{21;21\} & -\,& q^3\{1^3;3\} & +\,& q^4\{4;1^4\} \\
+\,& q^4\{31;21^2\} & +\,& q^4\{2^2;2^2\} & +\,& q^4\{21^2;31\} & +\,& q^4\{1^4;4\} \\
-\,& q^5\{5;1^5\} & -\,& q^5\{41;21^3\} & -\,& q^5\{32;2^2 1\} & -\,& q^5\{31^2;31^2\} \\
-\,& q^5\{2^2 1;32\} & -\,& q^5\{21^3;41\} & -\,& q^5\{1^5;5\} & +\,& q^6\{6;1^6\} \\
+\,& q^6\{51;21^4\} & +\,& q^6\{42;2^2 1^2\} & +\,& q^6\{3^2;2^3\} & +\,& q^6\{41^2;31^3\} \\
+\,& q^6\{321;321\} & +\,& q^6\{2^3;3^2\} & +\,& q^6\{31^3;41^2\} & +\,& q^6\{2^2 1^2;42\} \\
+\,& q^6\{21^4;51\} & +\,& q^6\{1^6;6\} & -\,& q^7\{7;1^7\} & -\,& q^7\{61;21^5\} \\
-\,& q^7\{52;2^2 1^3\} & -\,& q^7\{43;2^3 1\} & -\,& q^7\{51^2;31^4\} & -\,& q^7\{421;321^2\} \\
-\,& q^7\{3^2 1;32^2\} & -\,& q^7\{32^2;3^2 1\} & -\,& q^7\{41^3;41^3\} & -\,& q^7\{321^2;421\} \\
-\,& q^7\{2^3 1;43\} & -\,& q^7\{31^4;51^2\} & -\,& q^7\{2^2 1^3;52\} & -\,& q^7\{21^5;61\} \\
-\,& q^7\{1^7;7\} & & & & & & \\
\end{aligned}
$$

$$H(q)^{-1} =$$

$$
\begin{aligned}
&\{0;0\} & +\,& q\{1;1\} & +\,& q^2\{0;0\} & +\,& 2q^2\{1;1\} \\
+\,& q^2\{1^2;1^2\} & +\,& q^2\{2;2\} & +\,& 2q^3\{0;0\} & +\,& 5q^3\{1;1\} \\
+\,& 2q^3\{1^2;1^2\} & +\,& 2q^3\{2;1^2\} & +\,& q^3\{1^3;1^3\} & +\,& 2q^3\{1^2;2\} \\
+\,& 2q^3\{2;2\} & +\,& q^3\{21;21\} & +\,& q^3\{3;3\} & +\,& 5q^4\{0;0\} \\
+\,& 12q^4\{1;1\} & +\,& 8q^4\{1^2;1^2\} & +\,& 5q^4\{2;1^2\} & +\,& 2q^4\{1^3;1^3\} \\
+\,& 2q^4\{21;1^3\} & +\,& q^4\{1^4;1^4\} & +\,& 5q^4\{1^2;2\} & +\,& 8q^4\{2;2\} \\
+\,& 2q^4\{1^3;21\} & +\,& 4q^4\{21;21\} & +\,& 2q^4\{3;21\} & +\,& q^4\{21^2;21^2\} \\
+\,& q^4\{2^2;2^2\} & +\,& 2q^4\{21;3\} & +\,& 2q^4\{3;3\} & +\,& q^4\{31;31\} \\
+\,& q^4\{4;4\} & +\,& 8q^5\{0;0\} & +\,& 29q^5\{1;1\} & +\,& 20q^5\{1^2;1^2\} \\
+\,& 18q^5\{2;1^2\} & +\,& 8q^5\{1^3;1^3\} & +\,& 8q^5\{21;1^3\} & +\,& q^5\{3;1^3\} \\
+\,& 2q^5\{1^4;1^4\} & +\,& 2q^5\{21^2;1^4\} & +\,& q^5\{1^5;1^5\} & +\,& 18q^5\{1^2;2\} \\
+\,& 20q^5\{2;2\} & +\,& 8q^5\{1^3;21\} & +\,& 17q^5\{21;21\} & +\,& 8q^5\{3;21\} \\
+\,& 2q^5\{1^4;21^2\} & +\,& 4q^5\{21^2;21^2\} & +\,& 2q^5\{2^2;21^2\} & +\,& 2q^5\{31;21^2\} \\
+\,& q^5\{21^3;21^3\} & +\,& 2q^5\{21^2;2^2\} & +\,& 2q^5\{2^2;2^2\} & +\,& 2q^5\{31;2^2\} \\
+\,& q^5\{2^2 1;2^2 1\} & +\,& q^5\{1^3;3\} & +\,& 8q^5\{21;3\} & +\,& 8q^5\{3;3\} \\
+\,& 2q^5\{21^2;31\} & +\,& 2q^5\{2^2;31\} & +\,& 4q^5\{31;31\} & +\,& 2q^5\{4;31\} \\
+\,& q^5\{31^2;31^2\} & +\,& q^5\{32;32\} & +\,& 2q^5\{31;4\} & +\,& 2q^5\{4;4\} \\
+\,& q^5\{41;41\} & +\,& q^5\{5;5\} & +\,& 20q^6\{0;0\} & +\,& 66q^6\{1;1\} \\
+\,& 58q^6\{1^2;1^2\} & +\,& 47q^6\{2;1^2\} & +\,& 24q^6\{1^3;1^3\} & +\,& 28q^6\{21;1^3\} \\
+\,& 8q^6\{3;1^3\} & +\,& 8q^6\{1^4;1^4\} & +\,& 8q^6\{21^2;1^4\} & +\,& 3q^6\{2^2;1^4\} \\
+\,& q^6\{31;1^4\} & +\,& 2q^6\{1^5;1^5\} & +\,& 2q^6\{21^3;1^5\} & +\,& q^6\{1^6;1^6\} \\
+\,& 47q^6\{1^2;2\} & +\,& 58q^6\{2;2\} & +\,& 28q^6\{1^3;21\} & +\,& 60q^6\{21;21\} \\
+\,& 28q^6\{3;21\} & +\,& 8q^6\{1^4;21^2\} & +\,& 20q^6\{21^2;21^2\} & +\,& 9q^6\{2^2;21^2\} \\
+\,& 12q^6\{31;21^2\} & +\,& q^6\{4;21^2\} & +\,& 2q^6\{1^5;21^3\} & +\,& 4q^6\{21^3;21^3\} \\
+\,& 2q^6\{2^2 1;21^3\} & +\,& 2q^6\{31^2;21^3\} & +\,& q^6\{21^4;21^4\} & +\,& 3q^6\{1^4;2^2\} \\
\end{aligned}
$$

$$H(q)^{-1}(contd)$$

$+\,9q^6\{21^2;2^2\}$	$+\,11q^6\{2^2;2^2\}$	$+\,9q^6\{31;2^2\}$	$+\,3q^6\{4;2^2\}$
$+\,2q^6\{21^3;2^21\}$	$+\,4q^6\{2^21;2^21\}$	$+\,2q^6\{31^2;2^21\}$	$+\,2q^6\{32;2^21\}$
$+\,q^6\{2^21^2;2^21^2\}$	$+\,q^6\{2^3;2^3\}$	$+\,8q^6\{1^3;3\}$	$+\,28q^6\{21;3\}$
$+\,24q^6\{3;3\}$	$+\,q^6\{1^4;31\}$	$+\,12q^6\{21^2;31\}$	$+\,9q^6\{2^2;31\}$
$+\,20q^6\{31;31\}$	$+\,8q^6\{4;31\}$	$+\,2q^6\{21^3;31^2\}$	$+\,2q^6\{2^21;31^2\}$
$+\,4q^6\{31^2;31^2\}$	$+\,2q^6\{32;31^2\}$	$+\,2q^6\{41;31^2\}$	$+\,q^6\{31^3;31^3\}$
$+\,2q^6\{2^21;32\}$	$+\,2q^6\{31^2;32\}$	$+\,4q^6\{32;32\}$	$+\,2q^6\{41;32\}$
$+\,q^6\{321;321\}$	$+\,q^6\{3^2;3^2\}$	$+\,q^6\{21^2;4\}$	$+\,3q^6\{2^2;4\}$
$+\,8q^6\{31;4\}$	$+\,8q^6\{4;4\}$	$+\,2q^6\{31^2;41\}$	$+\,2q^6\{32;41\}$
$+\,4q^6\{41;41\}$	$+\,2q^6\{5;41\}$	$+\,q^6\{41^2;41^2\}$	$+\,q^6\{42;42\}$
$+\,2q^6\{41;5\}$	$+\,2q^6\{5;5\}$	$+\,q^6\{51;51\}$	$+\,q^6\{6;6\}$
$+\,34q^7\{0;0\}$	$+\,152q^7\{1;1\}$	$+\,144q^7\{1^2;1^2\}$	$+\,132q^7\{2;1^2\}$
$+\,70q^7\{1^3;1^3\}$	$+\,92q^7\{21;1^3\}$	$+\,27q^7\{3;1^3\}$	$+\,24q^7\{1^4;1^4\}$
$+\,32q^7\{21^2;1^4\}$	$+\,10q^7\{2^2;1^4\}$	$+\,10q^7\{31;1^4\}$	$+\,8q^7\{1^5;1^5\}$
$+\,8q^7\{21^3;1^5\}$	$+\,3q^7\{2^21;1^5\}$	$+\,q^7\{31^2;1^5\}$	$+\,q^7\{1^6;1^6\}$
$+\,2q^7\{21^4;1^6\}$	$+\,132q^7\{1^2;2\}$	$+\,144q^7\{2;2\}$	$+\,92q^7\{1^3;21\}$
$+\,189q^7\{21;21\}$	$+\,92q^7\{3;21\}$	$+\,32q^7\{1^4;21^2\}$	$+\,76q^7\{21^2;21^2\}$
$+\,42q^7\{2^2;21^2\}$	$+\,52q^7\{31;21^2\}$	$+\,10q^7\{4;21^2\}$	$+\,8q^7\{1^5;21^3\}$
$+\,20q^7\{21^3;21^3\}$	$+\,12q^7\{2^21;21^3\}$	$+\,12q^7\{31^2;21^3\}$	$+\,4q^7\{32;21^3\}$
$+\,q^7\{41;21^3\}$	$+\,2q^7\{1^6;21^4\}$	$+\,4q^7\{21^4;21^4\}$	$+\,2q^7\{2^21^2;21^4\}$
$+\,2q^7\{31^3;21^4\}$	$+\,q^7\{21^5;21^5\}$	$+\,10q^7\{1^4;2^2\}$	$+\,42q^7\{21^2;2^2\}$
$+\,34q^7\{2^2;2^2\}$	$+\,42q^7\{31;2^2\}$	$+\,10q^7\{4;2^2\}$	$+\,3q^7\{1^5;2^21\}$
$+\,12q^7\{21^3;2^21\}$	$+\,20q^7\{2^21;2^21\}$	$+\,13q^7\{31^2;2^21\}$	$+\,12q^7\{32;2^21\}$
$+\,4q^7\{41;2^21\}$	$+\,2q^7\{21^4;2^21^2\}$	$+\,4q^7\{2^21^2;2^21^2\}$	$+\,2q^7\{2^3;2^21^2\}$
$+\,2q^7\{31^3;2^21^2\}$	$+\,2q^7\{321;2^21^2\}$	$+\,q^7\{2^21^3;2^21^3\}$	$+\,2q^7\{2^21^2;2^3\}$
$+\,2q^7\{2^3;2^3\}$	$+\,2q^7\{321;2^3\}$	$+\,q^7\{2^31;2^31\}$	$+\,27q^7\{1^3;3\}$
$+\,92q^7\{21;3\}$	$+\,70q^7\{3;3\}$	$+\,10q^7\{1^4;31\}$	$+\,52q^7\{21^2;31\}$
$+\,42q^7\{2^2;31\}$	$+\,76q^7\{31;31\}$	$+\,32q^7\{4;31\}$	$+\,q^7\{1^5;31^2\}$
$+\,12q^7\{21^3;31^2\}$	$+\,13q^7\{2^21;31^2\}$	$+\,23q^7\{31^2;31^2\}$	$+\,13q^7\{32;31^2\}$
$+\,12q^7\{41;31^2\}$	$+\,q^7\{5;31^2\}$	$+\,2q^7\{21^4;31^3\}$	$+\,2q^7\{2^21^2;31^3\}$
$+\,4q^7\{31^3;31^3\}$	$+\,2q^7\{321;31^3\}$	$+\,2q^7\{41^2;31^3\}$	$+\,q^7\{31^4;31^4\}$
$+\,4q^7\{21^3;32\}$	$+\,12q^7\{2^21;32\}$	$+\,13q^7\{31^2;32\}$	$+\,20q^7\{32;32\}$
$+\,12q^7\{41;32\}$	$+\,3q^7\{5;32\}$	$+\,2q^7\{2^21^2;321\}$	$+\,2q^7\{2^3;321\}$
$+\,2q^7\{31^3;321\}$	$+\,6q^7\{321;321\}$	$+\,2q^7\{3^2;321\}$	$+\,2q^7\{41^2;321\}$
$+\,2q^7\{42;321\}$	$+\,q^7\{321;321^2\}$	$+\,q^7\{32^2;32^2\}$	$+\,2q^7\{321;3^2\}$
$+\,2q^7\{3^2;3^2\}$	$+\,2q^7\{42;3^2\}$	$+\,q^7\{3^21;3^21\}$	$+\,10q^7\{21^2;4\}$
$+\,10q^7\{2^2;4\}$	$+\,32q^7\{31;4\}$	$+\,24q^7\{4;4\}$	$+\,q^7\{21^3;41\}$
$+\,4q^7\{2^21;41\}$	$+\,12q^7\{31^2;41\}$	$+\,12q^7\{32;41\}$	$+\,20q^7\{41;41\}$
$+\,8q^7\{5;41\}$	$+\,2q^7\{31^3;41^2\}$	$+\,2q^7\{321;41^2\}$	$+\,4q^7\{41^2;41^2\}$
$+\,2q^7\{42;41^2\}$	$+\,2q^7\{51;41^2\}$	$+\,q^7\{41^3;41^3\}$	$+\,2q^7\{321;42\}$
$+\,2q^7\{3^2;42\}$	$+\,2q^7\{41^2;42\}$	$+\,4q^7\{42;42\}$	$+\,2q^7\{51;42\}$
$+\,q^7\{421;421\}$	$+\,q^7\{43;43\}$	$+\,q^7\{31^2;5\}$	$+\,3q^7\{32;5\}$
$+\,8q^7\{41;5\}$	$+\,8q^7\{5;5\}$	$+\,2q^7\{41^2;51\}$	$+\,2q^7\{42;51\}$
$+\,4q^7\{51;51\}$	$+\,2q^7\{6;51\}$	$+\,q^7\{51^2;51^2\}$	$+\,q^7\{52;52\}$
$+\,2q^7\{51;6\}$	$+\,2q^7\{6;6\}$	$+\,q^7\{61;61\}$	$+\,q^7\{7;7\}$